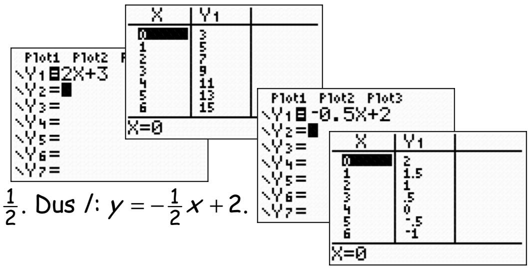


1 Ga je 1 naar rechts, dan kom je (op de lijn) 2 hoger uit.
Het getal 3 geeft aan dat de lijn de y-as in het punt (0, 3) snijdt.

2 Stel l: $y = ax + b$; het snijpunt met de y-as is (0, 2) $\Rightarrow b = 2$.
2 naar rechts dan 1 omlaag $\Rightarrow$ 1 naar rechts dan $\frac{1}{2}$ omlaag $\Rightarrow a = -\frac{1}{2}$. Dus l: $y = -\frac{1}{2}x + 2$.

3a Stel l: $y = ax + b$ met $a = \text{rc}_l = -2$.
l: $y = -2x + b$ door $B(-2, 3)$ $\Rightarrow 3 = -2 \cdot -2 + b$
$3 = 4 + b$
$-1 = b$.
Dus l: $y = -2x - 1$.

3b Stel k: $y = ax + b$ met $a = \text{rc}_k = \text{rc}_m = 4$.
k: $y = 4x + b$ door $B(-5, 21)$ $\Rightarrow 21 = 4 \cdot -5 + b$
$21 = -20 + b$
$41 = b$.
Dus k: $y = 4x + 41$.

4a Stel p: $y = ax + b$ met $a = \text{rc}_p = \text{rc}_q = -\frac{1}{3}$.
p: $y = -\frac{1}{3}x + b$ door $C(-18, 30)$ $\Rightarrow 30 = -\frac{1}{3} \cdot -18 + b$
$30 = 6 + b$
$24 = b$.
Dus p: $y = -\frac{1}{3}x + 24$.

4b p: $y = -\frac{1}{3}x + 24$ snijden met de x-as ($y = 0$) geeft:
$0 = -\frac{1}{3}x + 24 \Rightarrow \frac{1}{3}x = 24$ (keer 3) $\Rightarrow x = 72$. Dus $S(72, 0)$.
p: $y = -\frac{1}{3}x + 24$ snijden met de y-as ($x = 0$) geeft:
$y = -\frac{1}{3} \cdot 0 + 24 = 0 + 24 = 24$. Dus snijpunt met de y-as (0, 24).

5a k: $y = ax + 10$ door $P(-20, 0)$
$0 = a \cdot -20 + 10$
$0 = -20a + 10$
$20a = 10$
$a = \frac{10}{20} = \frac{1}{2}$.

5b k: $y = ax + 10$ door $Q(2, -4)$
$-4 = a \cdot 2 + 10$
$-4 = 2a + 10$
$-14 = 2a$
$-7 = a$.

5c k: $y = ax + 10$ door $O(0, 0)$
$0 = a \cdot 0 + 10$
$0 = 0 + 10$ (kan niet).
Er is dus geen a mogelijk.

6a m: $y = -2x + b$ door $P(-8, 0)$
$0 = -2 \cdot -8 + b$
$0 = 16 + b$
$-16 = b$.

6b m en l evenwijdig $\Rightarrow \text{rc}_m = \text{rc}_l = a = -2$
$y = -2x + b$ door $Q(10, 7)$
$7 = -2 \cdot 10 + b$
$27 = b$.

6c $R(8, 6)$ ligt op k, want $6 = 0{,}5 \cdot 8 + 2$.
l: $y = ax - 4$ door $R(8, 6)$
$6 = a \cdot 8 - 4$
$10 = 8a$
$\frac{10}{8} = \frac{5}{4} = a$.
m: $y = -2x + b$ door $R(8, 6)$
$6 = -2 \cdot 8 + b$
$6 = -16 + b$
$22 = b$.

6d k: $y = 0{,}5x + 2$ snijden met de x-as ($y = 0$)
$0 = 0{,}5x + 2$
$-2 = 0{,}5x$ (keer 2)
$-4 = x$.
Dus $S(-4, 0)$.
l: $y = ax - 4$ door $S(-4, 0)$
$0 = a \cdot -4 - 4$
$0 = -4a - 4$
$4a = -4$
$a = -1$.
m: $y = -2x + b$ door $S(-4, 0)$
$0 = -2 \cdot -4 + b$
$0 = 8 + b$
$-8 = b$.

7a ga je 4 naar rechts, dan ga je 3 omhoog,
dus ga je 1 ($= \frac{1}{4} \times 4$) naar rechts, dan ga je $\frac{3}{4}$ ($= \frac{1}{4} \times 3$) omhoog. Dus $\text{rc}_l = \frac{3}{4}$.

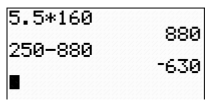
7b $\text{rc}_l = \frac{3}{4} = \frac{y_B - y_A}{x_B - x_A}$.

8a l: $y = ax + b$ met $a = \frac{\Delta y}{\Delta x} = \frac{4 - 1}{1 - -1} = \frac{3}{2} = 1{,}5$.
l: $y = 1{,}5x + b$ door $B(1, 4)$ $\Rightarrow 4 = 1{,}5 \cdot 1 + b$
$4 = 1{,}5 + b$
$2{,}5 = b$.
Dus l: $y = 1{,}5x + 2{,}5$.

8b k: $y = ax + b$ met $a = \frac{\Delta y}{\Delta x} = \frac{0 - 5}{2 - -3} = \frac{-5}{5} = -1$.
k: $y = -x + b$ door $D(2, 0)$ $\Rightarrow 0 = -2 + b$
$2 = b$.
Dus k: $y = -x + 2$.

8c m: $y = ax + b$ met $a = \frac{\Delta y}{\Delta x} = \frac{3 - 3}{-7 - 5} = \frac{0}{...} = 0$.
m: $y = b$ door $E(5, 3)$ $\Rightarrow 3 = b$
Dus m: $y = 3$.

8d n: $y = ax + b$ met $a = \frac{\Delta y}{\Delta x} = \frac{250 - 360}{160 - 180} = \frac{-110}{-20} = 5{,}5$.
n: $y = 5{,}5x + b$ door $H(160, 250)$ $\Rightarrow 250 = 5{,}5 \cdot 160 + b$
$250 = 880 + b$
$-630 = b$.
Dus n: $y = 5{,}5x - 630$.



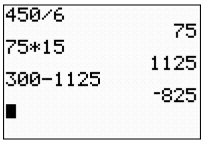
9a $A = as + b$ met $a = \frac{\Delta A}{\Delta s} = \frac{750-300}{21-15} = \frac{450}{6} = 75.$

$\left.\begin{array}{l} A = 75s + b \\ s = 15 \ \wedge \ A = 300 \end{array}\right\} \Rightarrow 300 = 75 \cdot 15 + b$

$300 = 1125 + b$

$-825 = b.$

Dus $A = 75s - 825.$



9b $R = at + b$ met $a = \frac{\Delta R}{\Delta t} = \frac{35-10}{60-35} = \frac{25}{25} = 1.$

$\left.\begin{array}{l} R = t + b \\ t = 35 \ \wedge \ R = 10 \end{array}\right\} \Rightarrow 10 = 35 + b$

$-25 = b.$

Dus $R = t - 25.$

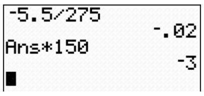
10a $p = aq + b$ met $a = \frac{\Delta p}{\Delta q} = \frac{2,25-7,75}{425-150} = \frac{-5,5}{275} = -0,02.$

$\left.\begin{array}{l} p = -0,02q + b \\ q = 150 \ \wedge \ p = 7,75 \end{array}\right\} \Rightarrow 7,75 = -0,02 \cdot 150 + b$

$7,75 = -3 + b$

$10,75 = b.$

Dus $p = -0,02q + 10,75.$



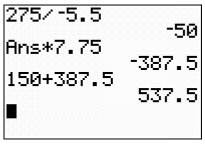
10b $q = ap + b$ met $a = \frac{\Delta q}{\Delta p} = \frac{425-150}{2,25-7,75} = \frac{275}{-5,5} = -50.$

$\left.\begin{array}{l} q = -50p + b \\ q = 150 \ \wedge \ p = 7,75 \end{array}\right\} \Rightarrow 150 = -50 \cdot 7,75 + b$

$150 = -387,5 + b$

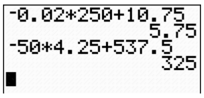
$537,5 = b.$

Dus $q = -50p + 537,5.$



10c $q = 250 \Rightarrow p = -0,02 \cdot 250 + 10,75 = 5,75.$

10d $p = 4,25 \Rightarrow q = -50 \cdot 4,25 + 537,5 = 325.$

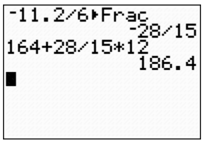


11a $h = at + b$ met $a = \frac{\Delta h}{\Delta t} = \frac{152,8-164,0}{18-12} = \frac{-11,2}{6} = -\frac{28}{15}.$

$\left.\begin{array}{l} h = -\frac{28}{15}t + b \\ t = 12 \ \wedge \ h = 164 \end{array}\right\} \Rightarrow 164 = -\frac{28}{15} \cdot 12 + b$

$186,4 = b.$

Dus $h = -\frac{28}{15}t + 186,4.$

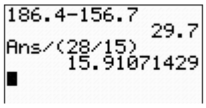


11b $h = 156,7$ geeft:

$156,7 = -\frac{28}{15}t + 186,4$

$\frac{28}{15}t = 186,4 - 156,7$

$t \approx 15,9.$ Dus om ongeveer 14:16.

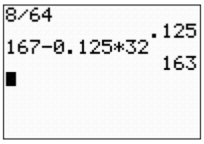


12a $L = at + b$ met $a = \frac{\Delta L}{\Delta t} = \frac{175-167}{96-32} = \frac{8}{64} = \frac{1}{8}.$

$\left.\begin{array}{l} L = \frac{1}{8}t + b \\ t = 32 \ \wedge \ L = 167 \end{array}\right\} \Rightarrow 167 = \frac{1}{8} \cdot 32 + b$

$163 = b.$

Dus $L = \frac{1}{8}t + 163.$

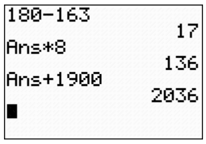


12b $L = 180$ geeft:

$180 = \frac{1}{8}t + 163$

$17 = \frac{1}{8}t$ (keer 8)

$136 = t$. Dus in 2036.

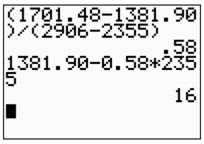


13a $B = ag + b$ met $a = \frac{\Delta B}{\Delta g} = \frac{1701,48-1381,90}{2906-2355} = 0,58.$

$\left.\begin{array}{l} B = 0,58g + b \\ g = 2355 \ \wedge \ B = 1381,90 \end{array}\right\} \Rightarrow 1381,90 = 0,58 \cdot 2355 + b$

$16 = b.$

Dus $B = 0,58g + 16.$

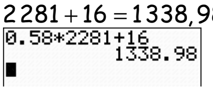


13bc Het vastrecht is 16 (€).

De prijs per m^3 is 0,58 (€).

$g = 2281$ (m^3) geeft

$B = 0,58 \cdot 2281 + 16 = 1338,98$ (€).

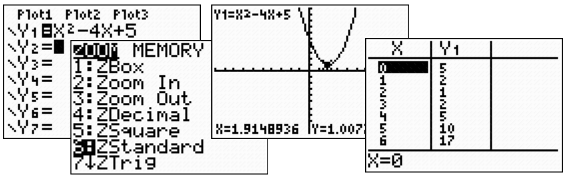


14a Zie de plot hiernaast.

14b De top is $(2, 1)$.

14c De top van een parabool ligt op de symmetrieas.

Dus $x_{top} = 2$. (in de tabel zie je dat $x = 2$ de symmetrieas is)

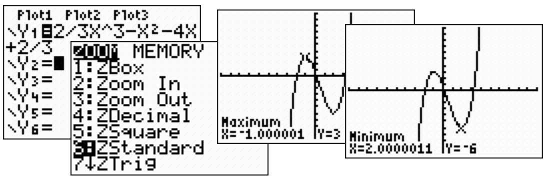


✲✲✲ **Neem GR - practicum 3 door.**

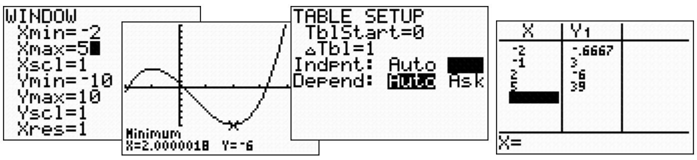
15a $f(x) = \frac{2}{3}x^3 - x^2 - 4x + \frac{2}{3}$ heeft

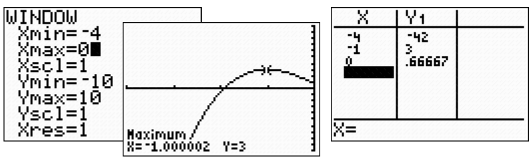
(optie maximum geeft) max. $f(-1) = 3$ en

(optie minimum geeft) min. $f(2) = -6$.



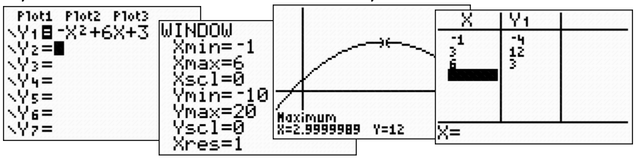
15b $D_f = [-2, 5]$ (plot op dit domein) $\Rightarrow B_f = [-6, 39]$. (gebruik 15a met de plot en de tabel hieronder)



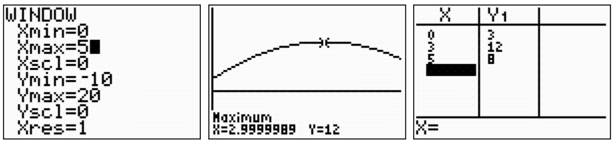


15c $D_f = [-4, 0]$ (plot op dit domein) $\Rightarrow B_f = [-42, 3]$. (gebruik 15a met de plot en de tabel hierboven)

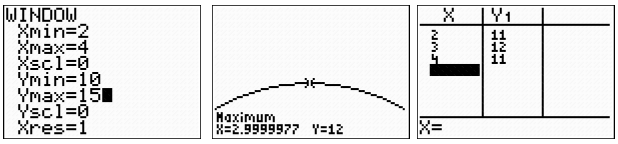
16a $f(x) = -x^2 + 6x + 3$ heeft (optie GR) max. $f(3) = 12$.
$D_f = [-1, 6]$ (plot op dit domein) $\Rightarrow B_f = [-4, 12]$.



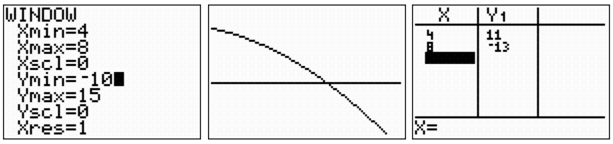
16b $D_f = [0, 5]$ (plot op dit domein) $\Rightarrow B_f = [3, 12]$.



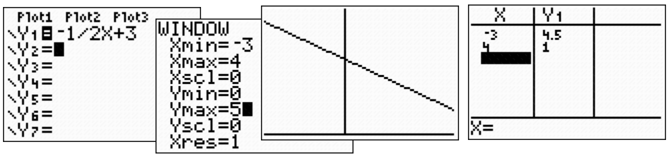
16c $D_f = [2, 4]$ (plot op dit domein) $\Rightarrow B_f = [11, 12]$.



16d $D_f = [4, 8]$ (plot op dit domein) $\Rightarrow B_f = [-13, 11]$.

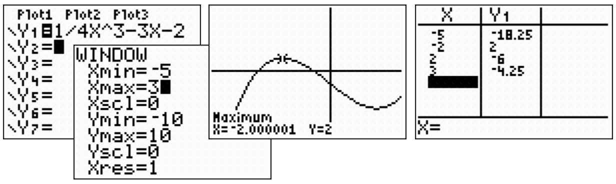


17 $f(x) = -\frac{1}{2}x + 3$ is een (rechte) lijn.
$D_f = [-3, 4]$ (plot op dit domein) $\Rightarrow B_f = [1, 4\frac{1}{2}]$.

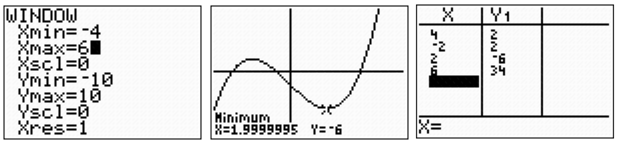


18a $f(x) = \frac{1}{4}x^3 - 3x - 2$ heeft (optie GR) max. $f(-2) = 2$ en min. $f(2) = -6$. (zie 18 b en 18 c)

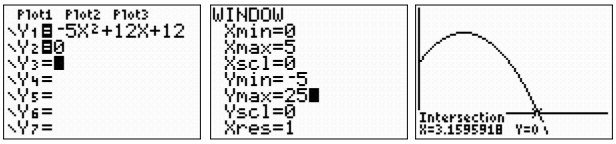
18b $D_f = [-5, 3]$ (plot op dit domein) $\Rightarrow B_f = [-18{,}25; 2]$.



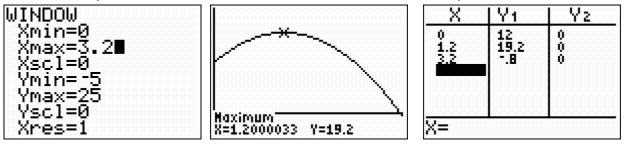
18c $D_f = [-4, 6]$ (plot op dit domein) $\Rightarrow B_f = [-6, 34]$.



19a $-5t^2 + 12t + 12 = 0$ (intersect) $\Rightarrow t \approx 3{,}2$. Dus $D_f = [0; 3{,}2]$.

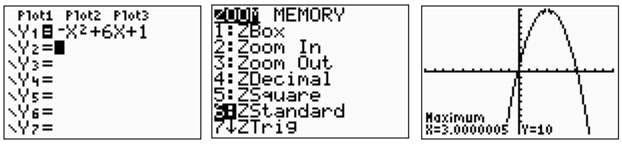


19b f heeft op dit domein max. $f(1{,}2) = 19{,}2$.
$D_f = [0; 3{,}2]$ (plot op dit domein) $\Rightarrow B_f = [0; 19{,}2]$.

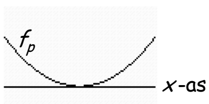


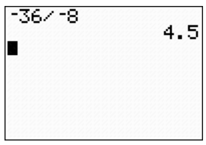
20a $p = 1 \Rightarrow f(x) = -x^2 + 6x + 1$ met top (3, 10). (optie maximum)

20b De grafiek van $f(x) = -x^2 + 6x + 1$ nu 10 eenheden naar beneden verschuiven dan komt de top op de x-as.
De formule wordt $f(x) = -x^2 + 6x - 9 \Rightarrow p = -9$.

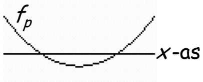


21a $f_p(x) = 2x^2 - 6x + p$ raakt de x-as als $D = 0$. ($2x^2 - 6x + p = 0$ heeft 1 oplossing)
$D = b^2 - 4ac = (-6)^2 - 4 \cdot 2 \cdot p = 36 - 8p = 0 \Rightarrow -8p = -36 \Rightarrow p = \frac{-36}{-8} = 4\frac{1}{2}$.

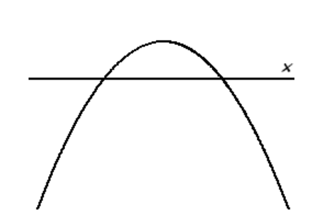
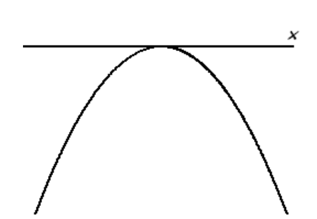
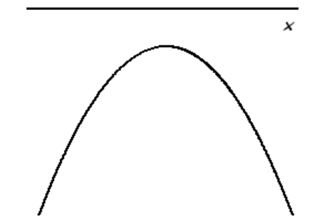




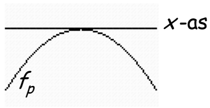
21b $f_p(x) = 2x^2 - 6x + p$ heeft een negatief minimum als $D = 36 - 8p > 0$.
(een parabool met de top onder de x-as snijdt de x-as in twee punten)
$D = 36 - 8p > 0 \Rightarrow -8p > -36$ (delen door $-8 \Rightarrow$ teken klapt om) $\Rightarrow p < 4\frac{1}{2}$.



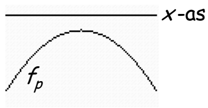
22

De grafiek van $y = ax^2 + bx + c$ met $a < 0$ ☹		
		
twee snijpunten met de x-as $D > 0$	één snijpunt met de x-as $D = 0$	geen snijpunten met de x-as $D < 0$

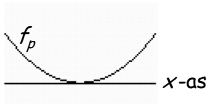
23a $D = b^2 - 4ac = (-5)^2 - 4 \cdot -\frac{1}{2} \cdot p = 0$
$25 + 2p = 0$
$2p = -25$
$p = -12\frac{1}{2}$.



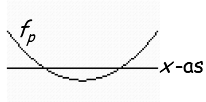
23b $D = 25 + 2p < 0$
$2p < -25$
$p < -12\frac{1}{2}$.



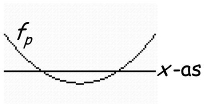
24a $D = b^2 - 4ac = p^2 - 4 \cdot 3 \cdot 3 = 0$
$p^2 - 36 = 0$
$p^2 = 36$
$p = -6 \vee p = 6$.



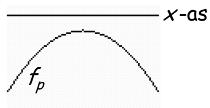
24b $D = p^2 - 36 > 0$
p^2 ☺ > 36
$p < -6 \vee p > 6$.



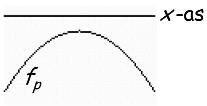
25a Minimum ☺ $\Rightarrow a > 0$ ☺ $\Rightarrow p > 0$. ❶
$D = b^2 - 4ac = (2p)^2 - 4 \cdot p \cdot 3 > 0$
$4p^2 - 12p$ ☺ > 0
$4p \cdot (p - 3) > 0$
$p < 0 \vee p > 3$. ❷
❶ én ❷ $\Rightarrow p > 3$.



25b Maximum ☹ $\Rightarrow a < 0$ ☹ $\Rightarrow p < 0$. ❶
$D = 4p^2 - 12p$ ☺ < 0
$4p \cdot (p - 3) < 0$
$0 < p < 3$. ❷
❶ én ❷ $\Rightarrow$ geen p voldoet.

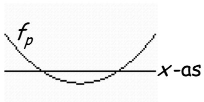


26a $D = b^2 - 4ac = p^2 - 4 \cdot -1 \cdot 2p < 0$
$p^2 + 8p$ ☺ < 0
$p \cdot (p + 8) < 0$
$-8 < p < 0$.



26b $f(2p) = -4$ geeft $-(2p)^2 + p \cdot 2p + 2p = -4$
$-4p^2 + 2p^2 + 2p = -4$
$-2p^2 + 2p + 4 = 0$
$p^2 - p - 2 = 0$
$(p - 2) \cdot (p + 1) = 0$
$p = 2 \vee p = -1$.

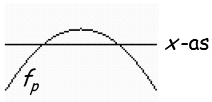
27a Minimum ☺ $\Rightarrow a > 0$ ☺ $\Rightarrow p > 0$. ❶
$D = b^2 - 4ac = (p + 2)^2 - 4 \cdot p \cdot 3 > 0$
$p^2 + 2p + 2p + 4 - 12p$ ☺ > 0
$p^2 - 8p + 4 > 0$ (zie hiernaast de berekening van de nulpunten)
$p < \frac{8 - \sqrt{48}}{2} \vee p > \frac{8 + \sqrt{48}}{2}$. ❷
❶ én ❷ $\Rightarrow 0 < p < \frac{8 - \sqrt{48}}{2} \vee p > \frac{8 + \sqrt{48}}{2}$.



$p^2 - 8p + 4 = 0$ ($a = 1$, $b = -8$ en $c = 4$)
$D^* = (-8)^2 - 4 \cdot 1 \cdot 4 = 64 - 16 = 48$
$p = \frac{-b \pm \sqrt{D^*}}{2a} = \frac{8 \pm \sqrt{48}}{2 \cdot 1} = \frac{8 \pm \sqrt{48}}{2}$
$p = \frac{8 + \sqrt{48}}{2} \vee p = \frac{8 - \sqrt{48}}{2}$.
(beide oplossingen zijn positief)

```
(8+√(48))/2
         7.464101615
(8-√(48))/2
         .5358983849
```

27b Maximum ☹ $\Rightarrow a < 0$ ☹ $\Rightarrow p < 0$. ❶
$D = p^2 - 8p + 4$ ☺ $> 0 \Rightarrow p < \frac{8 - \sqrt{48}}{2} \vee p > \frac{8 + \sqrt{48}}{2}$ (zie 27a). ❷
❶ én ❷ $\Rightarrow p < 0$.



28a $f(x) = ax^2 + bx = 0$
$x \cdot (ax + b) = 0$
$x = 0 \vee ax + b = 0$
$x = 0 \vee ax = -b$
$x = 0 \vee x = -\frac{b}{a}$.
(midden tussen deze x-waarden loopt de symmetrieas, dus)
$x_{top} = \frac{0 + -\frac{b}{a}}{2} = \frac{-\frac{b}{a}}{2} = \frac{1}{2} \cdot -\frac{b}{a} = -\frac{b}{2a}$.

28b $f(x) = ax^2 + bx + c = 0$ (abc-fomule)
$x = \frac{-b \pm \sqrt{D}}{2a}$
$x = \frac{-b + \sqrt{D}}{2a} \vee x = \frac{-b - \sqrt{D}}{2a}$.
$x_{top} = \frac{\frac{-b + \sqrt{D}}{2a} + \frac{-b - \sqrt{D}}{2a}}{2} = \frac{\frac{-b + \sqrt{D} - b - \sqrt{D}}{2a}}{2} = \frac{\frac{-2b}{2a}}{2} = \frac{1}{2} \cdot \frac{-2b}{2a} = -\frac{b}{2a}$.

ANDERS: $y = ax^2 + bx \xrightarrow[\text{omhoog verschuiven}]{c \text{ eenheden}} y = ax^2 + bx + c$.
Een verticale verschuiving verandert x_{top} niet.

29 $x_{top} = -\frac{b}{2a} = -\frac{p}{2 \cdot -2} = +\frac{p}{4} = \frac{1}{4}p$. ($a = -2 < 0$ ☹ $\Rightarrow$ inderdaad een maximum ☹)
Max. $y_{top} = f(x_{top}) = f(\frac{1}{4}p) = -2 \cdot (\frac{1}{4}p)^2 + p \cdot \frac{1}{4}p + 1 = -2 \cdot \frac{1}{16}p^2 + \frac{1}{4}p^2 + 1 = -\frac{1}{8}p^2 + \frac{2}{8}p^2 + 1 = \frac{1}{8}p^2 + 1 = 9$.
$\frac{1}{8}p^2 = 8$ (keer 8)
$p^2 = 64$
$p = -8 \vee p = 8$.

30 $x_{top} = -\frac{b}{2a} = -\frac{p}{2 \cdot 1} = -\frac{p}{2} = -\frac{1}{2}p.$

Max. $y_{top} = f(x_{top}) = f(-\frac{1}{2}p) = (-\frac{1}{2}p)^2 + p \cdot -\frac{1}{2}p + 3 = \frac{1}{4}p^2 - \frac{1}{2}p^2 + 3 = -\frac{1}{4}p^2 + 3.$

De top $(-\frac{1}{2}p,\ -\frac{1}{4}p^2 + 3)$ invullen in $y = x + 1$ geeft: $-\frac{1}{4}p^2 + 3 = -\frac{1}{2}p + 1$ (keer -4)

$p^2 - 12 = 2p - 4$

$p^2 - 2p - 8 = 0$

$(p - 4) \cdot (p + 2) = 0$

$p = 4 \ \vee\ p = -2.$

31a $x_{top} = -\frac{b}{2a} = -\frac{6}{2 \cdot p} = -\frac{6}{2p} = -\frac{3}{p}.$

$y_{top} = f(x_{top}) = f(-\frac{3}{p}) = p \cdot (-\frac{3}{p})^2 + 6 \cdot -\frac{3}{p} + 1 = \frac{p}{1} \cdot \frac{9}{p^2} - \frac{18}{p} + 1 = \frac{9}{p} - \frac{18}{p} + 1 = -\frac{9}{p} + 1 = -2.$

$\frac{-9}{p} = -3 = \frac{-3}{1}$ (kruiselings vermenigvuldgen)

$-3p = -9$

$p = 3.$

31b $a = p = 3 > 0$ ☺ $\Rightarrow$ de extreme waarde is een minimum ☺.

32 $x_{top} = -\frac{b}{2a} = -\frac{p+2}{2 \cdot p} = -\frac{p+2}{2p}.$ ❶

$y_{top} = f(-\frac{p+2}{2p}) = p \cdot (-\frac{p+2}{2p})^2 + (p+2) \cdot -\frac{p+2}{2p} + 5 = \frac{p}{1} \cdot \frac{(p+2)^2}{4p^2} - \frac{(p+2)^2}{2p} + 5 = \frac{(p+2)^2}{4p} - \frac{2(p+2)^2}{4p} + 5 = -\frac{(p+2)^2}{4p} + 5 = 3.$

$\frac{-(p+2)^2}{4p} = -2 = \frac{2}{-1}$ (kruiselings vermenigvuldgen)

$(p+2)^2 = 8p$

$p^2 + 2p + 2p + 4 = 8p$

$p^2 - 4p + 4 = 0$

$(p - 2) \cdot (p - 2) = 0$

$p = 2.$ ❷ Nu nog ❷ in ❶ $\Rightarrow x_{top} = -\frac{2+2}{2 \cdot 2} = -1$

33 $x_{top} = -\frac{b}{2a} = -\frac{p-4}{2 \cdot p} = -\frac{p-4}{2p}.$

Max. $y_{top} = f(x_{top}) = f(-\frac{p-4}{2p}) = p \cdot (-\frac{p-4}{2p})^2 + (p-4) \cdot -\frac{p-4}{2p} + 3 = \frac{(p-4)^2}{4p} - \frac{(p-4)^2}{2p} \cdot \frac{2}{2} + 3 = -\frac{(p-4)^2}{4p} + 3.$

De top $(-\frac{p-4}{2p},\ -\frac{(p-4)^2}{4p} + 3)$ invullen in $y = x + 9$ geeft: $-\frac{(p-4)^2}{4p} + 3 = -\frac{p-4}{2p} + 9$ (intersect of)

$-\frac{p^2 - 4p - 4p + 16}{4p} + \frac{2 \cdot (p-4)}{2 \cdot 2p} = 6$

$\frac{-p^2 + 4p + 4p - 16}{4p} + \frac{2p - 8}{4p} = 6$

$\frac{-p^2 + 10p - 24}{4p} = \frac{6}{1}$

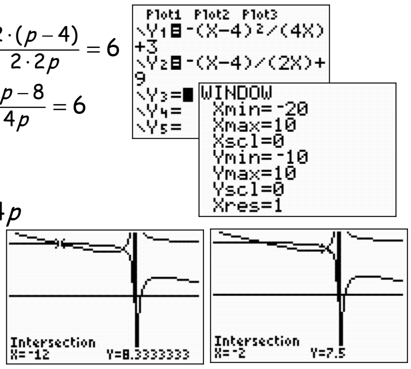
$-p^2 + 10p - 24 = 24p$

$-p^2 - 14p - 24 = 0$

$p^2 + 14p + 24 = 0$

$(p + 12) \cdot (p + 2) = 0$

$p = -12 \ \vee\ p = -2.$



$p = -12 \Rightarrow \begin{cases} x_{top} = -\frac{p-4}{2p} = -\frac{-16}{-24} = -\frac{2}{3} \\ y_{top} = f(-\frac{2}{3}) = -\frac{2}{3} + 9 = 8\frac{1}{3} \text{ (extreem).} \end{cases}$

$p = -2 \Rightarrow \begin{cases} x_{top} = -\frac{p-4}{2p} = -\frac{-6}{-4} = -1\frac{1}{2} \\ y_{top} = f(-1\frac{1}{2}) = -1\frac{1}{2} + 9 = 7\frac{1}{2} \text{ (extreem).} \end{cases}$

Met intersect zijn de extremen (Y) direct af te lezen.

34a Zie de gevraagde plot hiernaast.

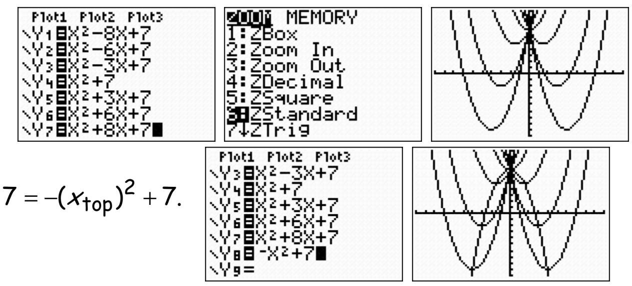
34b $x_{top} = -\frac{b}{2a} = -\frac{p}{2 \cdot 1} = -\frac{p}{2} = -\frac{1}{2}p$ (keer -2) $\Rightarrow -2 \cdot x_{top} = p.$

$y_{top} = f(x_{top}) \Rightarrow y_{top} = (x_{top})^2 + p \cdot x_{top} + 7$ ❶

$p = -2 \cdot x_{top}$ invullen in ❶ geeft dan

$y_{top} = (x_{top})^2 + -2 \cdot x_{top} \cdot x_{top} + 7 = (x_{top})^2 - 2 \cdot (x_{top})^2 + 7 = -(x_{top})^2 + 7.$

34c Zie de gevraagde plot hiernaast.



💻

35 Voor de x-coördinaat van de toppen geldt: $x = -\frac{b}{2a} = -\frac{p}{2 \cdot -\frac{1}{8}} = \frac{p}{\frac{1}{4}} \cdot \frac{4}{4} = 4p \Rightarrow \frac{1}{4}x = p \Rightarrow p = \frac{1}{4}x.$

$y = f(x) \Rightarrow y = -\frac{1}{8}x^2 + \frac{1}{4}x \cdot x - 6 = -\frac{1}{8}x^2 + \frac{2}{8} \cdot x^2 - 6 = \frac{1}{8} \cdot x^2 - 6.$ Dus de toppen liggen op $y = \frac{1}{8}x^2 - 6.$

36 Voor de x-coördinaat van de toppen geldt: $x = -\frac{b}{2a} = -\frac{6}{2\cdot p} = -\frac{3}{p} \Rightarrow px = -3 \Rightarrow p = -\frac{3}{x}$.
$y = f(x) \Rightarrow y = -\frac{3}{x}\cdot x^2 + 6\cdot x - \frac{3}{x} = -3x + 6x - \frac{3}{x} = 3x - \frac{3}{x}$. Dus de toppen liggen op $y = 3x - \frac{3}{x}$.

37 Voor de x-coördinaat van de toppen geldt: $x = -\frac{b}{2a} = -\frac{p}{2\cdot -1} = \frac{p}{2} \Rightarrow p = 2x$.
$y = f(x) \Rightarrow y = -x^2 + 2x\cdot x + 2\cdot 2x = -x^2 + 2x^2 + 4x = x^2 + 4x$. Dus de toppen liggen op $y = x^2 + 4x$.

38 Voor de x-coördinaat van de toppen geldt: $x = -\frac{b}{2a} = -\frac{-2p}{2\cdot p^2} = \frac{1}{p} \Rightarrow px = 1 \Rightarrow p = \frac{1}{x}$.
$y = f(x) \Rightarrow y = (\frac{1}{x})^2\cdot x^2 - 2\cdot\frac{1}{x}\cdot x + 3 = 1 - 2 + 3 = 2$. Dus de toppen liggen op $y = 2$ (een horizontale lijn).

39 Voor de x-coördinaat van de toppen geldt: $x = -\frac{b}{2a} = -\frac{-p}{2\cdot p} = \frac{1}{2}$. Dus de toppen liggen op $x = \frac{1}{2}$ (een verticale lijn).

40a Voor de x-coördinaat van de toppen geldt: $x = -\frac{b}{2a} = -\frac{1}{2\cdot p} = -\frac{1}{2p}$. (nu een vergelijking in p zoeken)
$y = f(x) \Rightarrow y = p\cdot(-\frac{1}{2p})^2 + (-\frac{1}{2p}) + \frac{1}{p} = p\cdot\frac{1}{4p^2} - \frac{1}{2p} + \frac{1}{p} = \frac{1}{4}\cdot\frac{1}{p} - \frac{1}{2}\cdot\frac{1}{p} + 1\cdot\frac{1}{p} = \frac{3}{4}\cdot\frac{1}{p} = \frac{3}{4p}$.
$y = \frac{3}{4p}$ moet $y = 6$ zijn $\Rightarrow \frac{3}{4p} = \frac{6}{1} \Rightarrow 24p = 3 \Rightarrow p = \frac{1}{8}$.

40b Voor de x-coördinaat van de toppen geldt: $x = -\frac{1}{2p}$ (zie 40a) $\Rightarrow 2px = -1 \Rightarrow p = -\frac{1}{2x}$.
$y = f(x) \Rightarrow y = -\frac{1}{2x}\cdot x^2 + x + -2x = -\frac{1}{2}x + x - 2x = -1\frac{1}{2}x$. Dus de toppen liggen op $y = -1\frac{1}{2}x$.

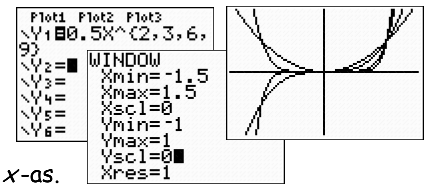
41a Voor de x-coördinaat van de toppen geldt: $x = -\frac{b}{2a} = -\frac{-10}{2\cdot p} = \frac{5}{p}$. (vergelijking in p zoeken)
$y = f(x) \Rightarrow y = p\cdot(\frac{5}{p})^2 - 10\cdot\frac{5}{p} + p + 3 = p\cdot\frac{25}{p^2} - \frac{50}{p} + p + 3 = \frac{25}{p} - \frac{50}{p} + p + 3 = -\frac{25}{p} + p + 3$.
$x = \frac{5}{p}$ en $y = -\frac{25}{p} + p + 3$ invullen in $y = -x - 5 \Rightarrow -\frac{25}{p} + p + 3 = -\frac{5}{p} - 5$ (uitwerken geeft)

$p + 3 + 5 = -\frac{5}{p} + \frac{25}{p}$
$\frac{p+8}{1} = \frac{20}{p}$ (kruiselings vermenigvuldigen)
$p\cdot(p+8) = 20$
$p^2 + 8p - 20 = 0$
$(p+10)\cdot(p-2) = 0$
$p = -10 \vee p = 2$.

$p = -10 \Rightarrow \begin{cases} x_{\text{top}} = \frac{5}{p} = \frac{5}{-10} = -\frac{1}{2} \\ y_{\text{top}} = f(-\frac{1}{2}) = --\frac{1}{2} - 5 = -4\frac{1}{2} \text{ (extreem).} \end{cases}$
$p = 2 \Rightarrow \begin{cases} x_{\text{top}} = \frac{5}{p} = \frac{5}{2} = 2\frac{1}{2} \\ y_{\text{top}} = f(2\frac{1}{2}) = -2\frac{1}{2} - 5 = -7\frac{1}{2} \text{ (extreem).} \end{cases}$

41b Voor de x-coördinaat van de toppen geldt: $x = -\frac{b}{2a} = -\frac{-10}{2\cdot p} = \frac{5}{p}$ (zie 41a) $\Rightarrow px = 5 \Rightarrow p = \frac{5}{x}$.
$y = f(x) \Rightarrow y = \frac{5}{x}\cdot x^2 - 10x + \frac{5}{x} + 3 = 5x - 10x + \frac{5}{x} + 3 = -5x + \frac{5}{x} + 3$. Dus de toppen liggen op $y = -5x + \frac{5}{x} + 3$.

42a Zie de plot hiernaast. (CALC en TABLE werken niet)
([,] is de toets boven [7]; accolades met [2nd][(] en [2nd][)])
(natuurlijk ook als vier verschillende formules in te voeren)

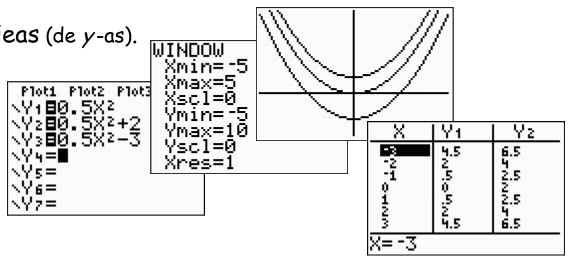


42b Alle grafieken gaan door $O(0, 0)$ en $(1; 0,5)$.

42c De grafieken van $y_1 = 0,5x^2$ en $y_3 = 0,5x^6$ komen niet onder de x-as.

42d De grafieken van $y_1 = 0,5x^2$ en $y_3 = 0,5x^6$ hebben een symmetrieas (de y-as).

43a Zie de plot hiernaast.



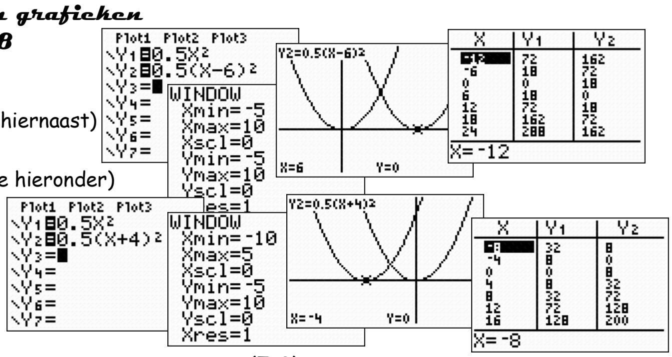
43b $y_1 = 0,5x^2$ ☺ $\xrightarrow[\text{(2 omhoog)}]{\text{translatie }(0,2)}$ $y_2 = 0,5x^2 + 2$.
Translatie $(0, 2)$ is een verschuiving van 0 naar rechts en 2 eenheden omhoog.

43c $y_1 = 0,5x^2$ ☺ $\xrightarrow[\text{(3 omlaag)}]{\text{translatie }(0,-3)}$ $y_3 = 0,5x^2 - 3$.

43d $y = 0,5x^2$ ☺ $\xrightarrow[\text{(6 omhoog)}]{\text{translatie }(0,6)}$ $y = 0,5x^2 + 6$.

44a $y_1 = 0{,}5x^2$ ☺ $\xrightarrow[\text{(6 naar rechts)}]{\text{translatie } (6,0)} y_2 = 0{,}5(x-6)^2$. (zie hiernaast)

44b $y_1 = 0{,}5x^2$ ☺ $\xrightarrow[\text{(4 naar links)}]{\text{translatie } (-4,0)} y_3 = 0{,}5(x+4)^2$. (zie hieronder)



44c $y = 0{,}5x^2$ ☺ $\xrightarrow[\text{(2 naar rechts)}]{\text{translatie } (2,0)} y = 0{,}5(x-2)^2$.

45a $y = -5x^2$ ☹ $\xrightarrow{\text{tr. } (2,5)} y = -5(x-2)^2 + 5$.

45b $y = -5x^2$ ☹ $\xrightarrow{\text{tr. } (-3,6)} y = -5(x+3)^2 + 6$.

45c $y = -5x^2$ ☹ $\xrightarrow{\text{tr. } (7,0)} y = -5(x-7)^2$.

46 $y = 2x^2$ ☺ $\xrightarrow{\text{tr. } (-2,0)} g(x) = 2(x+2)^2$.
$y = 2x^2$ ☺ $\xrightarrow{\text{tr. } (2,-2)} h(x) = 2(x-2)^2 - 2$.
$y = 2x^2$ ☺ $\xrightarrow{\text{tr. } (-1,-3)} k(x) = 2(x+1)^2 - 3$.
$y = 2x^2$ ☺ $\xrightarrow{\text{tr. } (1,-4)} l(x) = 2(x-1)^2 - 4$.

47a $y = -3x^2$ ☹ $\xrightarrow{\text{tr. } (0,2)} f(x) = -3x^2 + 2$.
$\begin{cases} \text{top } (0,0) \\ \text{max. } y(0) = 0 \\ B = \langle \leftarrow, 0] \end{cases} \Rightarrow \begin{cases} \text{top } (0,2) \\ \text{max. } f(0) = 2 \\ B = \langle \leftarrow, 2] \end{cases}$

47b $y = -3x^4$ ☹ $\xrightarrow{\text{tr. } (2,8)} g(x) = -3(x-2)^4 + 8$.
$\begin{cases} \text{top } (0,0) \\ \text{max. } y(0) = 0 \\ B = \langle \leftarrow, 0] \end{cases} \Rightarrow \begin{cases} \text{top } (2,8) \\ \text{max. } g(2) = 8 \\ B = \langle \leftarrow, 8] \end{cases}$

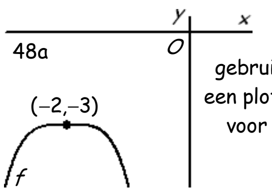
47c $y = 5x^2$ ☺ $\xrightarrow{\text{tr. } (-1,0)} h(x) = 5(x+1)^2$.
$\begin{cases} \text{top } (0,0) \\ \text{min. } y(0) = 0 \\ B = [0, \rightarrow\rangle \end{cases} \Rightarrow \begin{cases} \text{top } (-1,0) \\ \text{min. } h(-1) = 0 \\ B = [0, \rightarrow\rangle \end{cases}$

47d $y = 5x^6$ ☺ $\xrightarrow{\text{tr. } (0,1)} k(x) = 5x^6 + 1$.
$\begin{cases} \text{top } (0,0) \\ \text{min. } y(0) = 0 \\ B = [0, \rightarrow\rangle \end{cases} \Rightarrow \begin{cases} \text{top } (0,1) \\ \text{min. } k(0) = 1 \\ B = [1, \rightarrow\rangle \end{cases}$

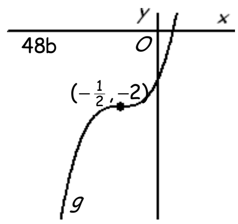
47e $y = -0{,}5x^2$ ☹ $\xrightarrow{\text{tr. } (100,0)} l(x) = -0{,}5(x-100)^2$.
$\begin{cases} \text{top } (0,0) \\ \text{max. } y(0) = 0 \\ B = \langle \leftarrow, 0] \end{cases} \Rightarrow \begin{cases} \text{top } (100,0) \\ \text{max. } l(100) = 0 \\ B = \langle \leftarrow, 0] \end{cases}$

47f $y = -0{,}4x^2$ ☹ $\xrightarrow{\text{tr. } (-0{,}1;\,-0{,}3)} m(x) = -0{,}4(x+0{,}1)^2 - 0{,}3$.
$\begin{cases} \text{top } (0,0) \\ \text{max. } y(0) = 0 \\ B = \langle \leftarrow, 0] \end{cases} \Rightarrow \begin{cases} \text{top } (-0{,}1;\,-0{,}3) \\ \text{max. } m(-0{,}1) = -0{,}3 \\ B = \langle \leftarrow; -0{,}3] \end{cases}$

48a $y = -2x^4$ ☹ $\xrightarrow{\text{tr. } (-2,-3)} f(x) = -2(x+2)^4 - 3$.
$\begin{cases} \text{top } (0,0) \\ \text{max. } y(0) = 0 \end{cases} \Rightarrow \begin{cases} \text{top } (-2,-3) \\ \text{max. } f(-2) = -3 \end{cases}$

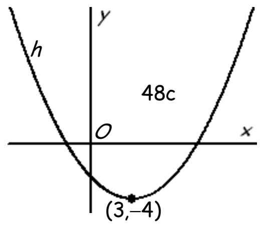


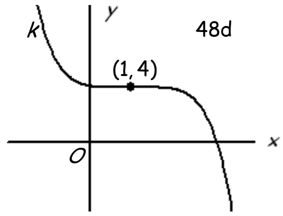
gebruik eventueel een plot en/of tabel voor de schets



48b $y = 6x^3$ ↗ $\xrightarrow{\text{tr. } (-\frac{1}{2},-2)} g(x) = 6(x+\frac{1}{2})^3 - 2$.
symm. in $(0,0)$ $\Rightarrow$ symmetrisch in $(-\frac{1}{2},-2)$

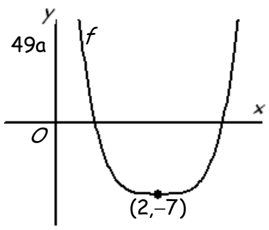
48c $y = 0{,}18x^2$ ☺ $\xrightarrow{\text{tr. } (3,-4)} h(x) = 0{,}18(x-3)^2 - 4$.
$\begin{cases} \text{top } (0,0) \\ \text{min. } y(0) = 0 \end{cases} \Rightarrow \begin{cases} \text{top } (3,-4) \\ \text{min. } h(3) = -4 \end{cases}$

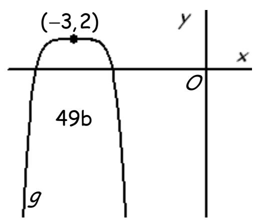




48d $y = -0{,}1x^5$ ↘ $\xrightarrow{\text{tr. } (1,4)} k(x) = -0{,}1(x-1)^5 + 4$.
symm. in $(0,0)$ $\Rightarrow$ symmetrisch in $(1,4)$

49a $y = 3x^4$ ☺ $\xrightarrow{\text{tr. } (2,-7)} f(x) = 3(x-2)^4 - 7$.
$\begin{cases} \text{top } (0,0) \\ \text{min. } y(0) = 0 \end{cases} \Rightarrow \begin{cases} \text{top } (2,-7) \\ \text{min. } f(2) = -7 \end{cases}$

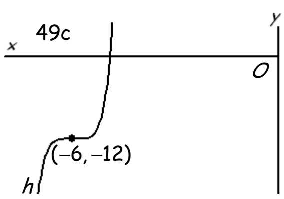


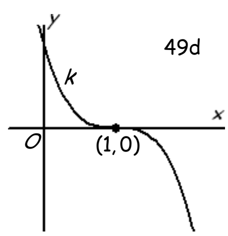


49b $y = -5x^6$ ☹ $\xrightarrow{\text{tr. } (-3,2)} g(x) = -5(x+3)^6 + 2$.
$\begin{cases} \text{top } (0,0) \\ \text{max. } y(0) = 0 \end{cases} \Rightarrow \begin{cases} \text{top } (-3,2) \\ \text{max. } g(-3) = -2 \end{cases}$

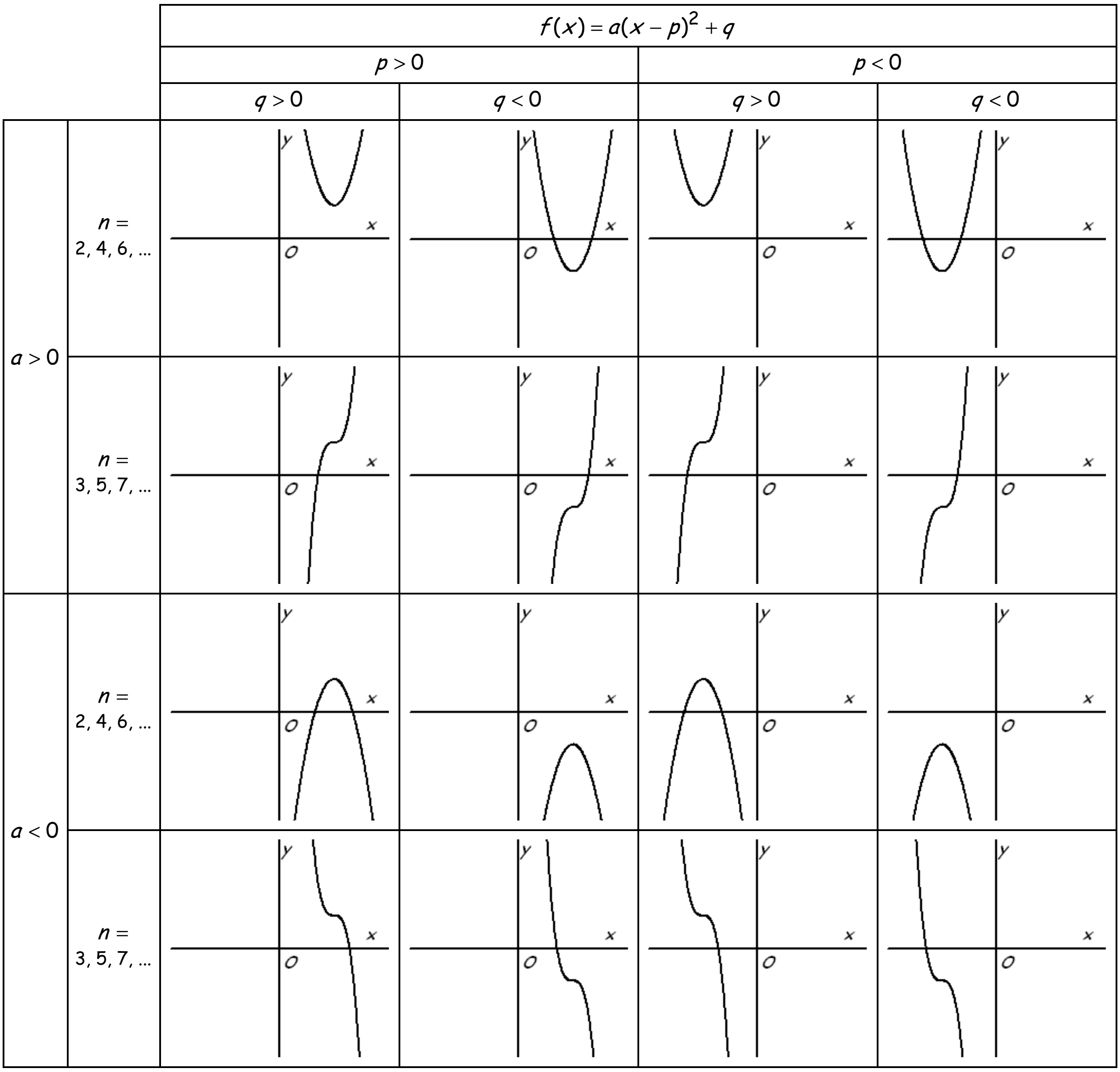
49c $y = 6x^5$ ↗ $\xrightarrow{\text{tr. } (-6,-12)} h(x) = 8(x+6)^5 - 12$.
symm. in $(0,0)$ $\Rightarrow$ symmetrisch in $(-6,-12)$

49d $y = -8x^3$ ↘ $\xrightarrow{\text{tr. } (1,0)} k(x) = -8(x-1)^3$.
symm. in $(0,0)$ $\Rightarrow$ symmetrisch in $(1,0)$

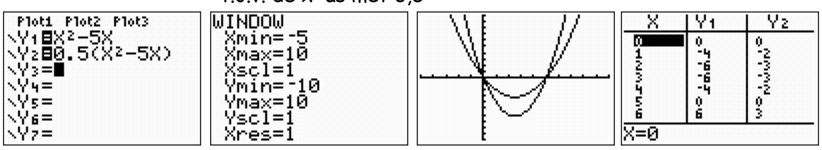




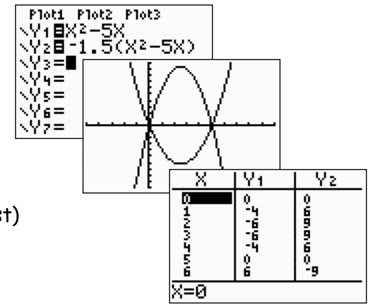
50



51a $y_1 = x^2 - 5x$ ☺ $\xrightarrow[\text{t.o.v. de } x\text{-as met } 0{,}5]{\text{vermenigvuldigen}}$ $y_2 = 0{,}5(x^2 - 5x)$. (zie de plot hieronder)



51b $y_1 = x^2 - 5x$ ☺ $\xrightarrow[\text{t.o.v. de } x\text{-as met } -1{,}5]{\text{vermenigvuldigen}}$ $y_3 = -1{,}5(x^2 - 5x)$ ☹. (zie de plot hiernaast)



52 $y = -0{,}5x^3$ ↘ $\xrightarrow{\text{tr. } (-3, -5)}$ $y = -0{,}5(x+3)^3 - 5$ ↘ $\xrightarrow{\text{verm. } x\text{-as, } -3}$ $y = 1{,}5(x+3)^3 + 15$ ↗.

53a $y = 0{,}5(x-3)^4 + 7$ ☺ $\xrightarrow{\text{tr. } (1, 2)}$ $y = 0{,}5(x-4)^4 + 9$ ☺ $\xrightarrow{\text{verm. } x\text{-as, } 1\frac{1}{2}}$ $y = 0{,}75(x-4)^4 + 13\frac{1}{2}$ ☺.
top (3, 7) ⇒ top (4, 9) ⇒ top $(4, 13\frac{1}{2})$

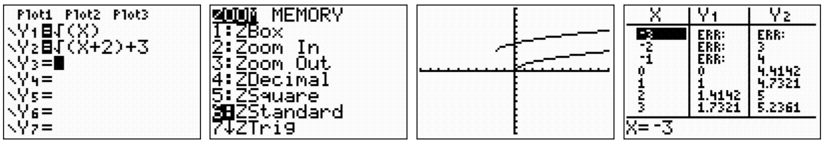
53b $y = -2{,}5(x+4)^6 - 7$ ☹ $\xrightarrow{\text{verm. } x\text{-as, } 2}$ $y = -5(x+4)^6 - 14$ ☹ $\xrightarrow{\text{tr. } (-1, 3)}$ $y = -5(x+5)^6 - 11$ ☹.
top (−4, −7) ⇒ top (−4, −14) ⇒ top (−5, −11)

54a $y=0{,}3x^4$ ☺ $\xrightarrow{\text{tr. }(-5,6)}$ $y=0{,}3(x+5)^4+6$ ☺ $\xrightarrow{\text{verm. }x\text{-as, }-3}$ $y=-0{,}9(x+5)^4-18$ ☹.
top (0, 0) $\Rightarrow$ top (–5, 6) $\Rightarrow$ top (–5, –18)

54b $y=0{,}3x^4$ ☺ $\xrightarrow{\text{verm. }x\text{-as, }-3}$ $y=-0{,}9x^4$ ☹ $\xrightarrow{\text{tr. }(-5,6)}$ $y=-0{,}9(x+5)^4+6$ ☹.
top (0, 0) $\Rightarrow$ top (0, 0) $\Rightarrow$ top (–5, 6)

55a Vermenigvuldigen t.o.v. de x-as met -1 komt op hetzelfde neer als spiegelen in de x-as.

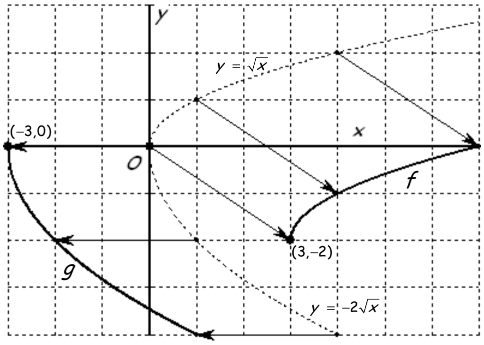
55b $y=3(x-1)^2-6$ ☺ $\xrightarrow[\text{(spiegelen in }x\text{-as)}]{\text{verm. }x\text{-as, }-1}$ $y=-3(x-1)^2+6$ ☹.



56ab $y=\sqrt{x}$ $\xrightarrow{\text{tr. }(-2,3)}$ $y=\sqrt{x+2}+3$.
$\begin{cases}D=[0,\rightarrow\rangle\\B=[0,\rightarrow\rangle\end{cases}$ $\Rightarrow$ $\begin{cases}D=[-2,\rightarrow\rangle\\B=[3,\rightarrow\rangle\end{cases}$

57ac $y=\sqrt{x}$ $\xrightarrow{\text{tr. }(3,-2)}$ $f(x)=\sqrt{x-3}-2$.
$\begin{cases}D=[0,\rightarrow\rangle\\B=[0,\rightarrow\rangle\end{cases}$ $\Rightarrow$ $\begin{cases}D_f=[3,\rightarrow\rangle\\B_f=[-2,\rightarrow\rangle\end{cases}$

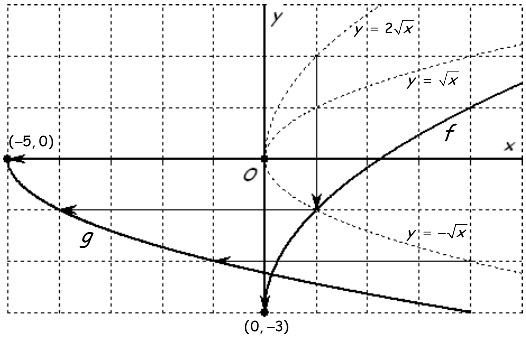
$y=\sqrt{x}$ $\xrightarrow{\text{verm. }x\text{-as, }-2}$ $y=-2\sqrt{x}$ $\xrightarrow{\text{tr. }(-3,0)}$ $g(x)=-2\sqrt{x+3}$.
$\begin{cases}D=[0,\rightarrow\rangle\\B=[0,\rightarrow\rangle\end{cases}$ $\Rightarrow$ $\begin{cases}D=[0,\rightarrow\rangle\\B=\langle\leftarrow,0]\end{cases}$ $\Rightarrow$ $\begin{cases}D_g=[-3,\rightarrow\rangle\\B_g=\langle\leftarrow,0]\end{cases}$



57b Zie de grafieken hiernaast. (een schets volstaat)

58ac $y=\sqrt{x}$ $\xrightarrow{\text{verm. }x\text{-as, }2}$ $y=2\sqrt{x}$ $\xrightarrow{\text{tr. }(0,-3)}$ $f(x)=2\sqrt{x}-3$.
$\begin{cases}D=[0,\rightarrow\rangle\\B=[0,\rightarrow\rangle\end{cases}$ $\Rightarrow$ $\begin{cases}D=[0,\rightarrow\rangle\\B=[0,\rightarrow\rangle\end{cases}$ $\Rightarrow$ $\begin{cases}D_f=[0,\rightarrow\rangle\\B_f=[-3,\rightarrow\rangle\end{cases}$

$y=\sqrt{x}$ $\xrightarrow{\text{verm. }x\text{-as, }-1}$ $y=-\sqrt{x}$ $\xrightarrow{\text{tr. }(-5,0)}$ $f(x)=-\sqrt{x+5}$.
$\begin{cases}D=[0,\rightarrow\rangle\\B=[0,\rightarrow\rangle\end{cases}$ $\Rightarrow$ $\begin{cases}D=[0,\rightarrow\rangle\\B=\langle\leftarrow,0]\end{cases}$ $\Rightarrow$ $\begin{cases}D_g=[-5,\rightarrow\rangle\\B_g=\langle\leftarrow,0]\end{cases}$



58b Zie de grafieken hiernaast. (een schets volstaat)

59a $y=\sqrt{x}$ $\xrightarrow{\text{tr. }(-5,3)}$ $f(x)=\sqrt{x+5}+3$.
$\begin{cases}\text{beginpunt }(0,0)\\D=[0,\rightarrow\rangle\\B=[0,\rightarrow\rangle\end{cases}$ $\Rightarrow$ $\begin{cases}\text{beginpunt }(-5,3)\\D_f=[-5,\rightarrow\rangle\\B_f=[3,\rightarrow\rangle\end{cases}$

59b $y=\sqrt{x}$ $\xrightarrow{\text{tr. }(-3,-7)}$ $g(x)=\sqrt{x+3}-7$.
$\begin{cases}\text{beginpunt }(0,0)\\D=[0,\rightarrow\rangle\\B=[0,\rightarrow\rangle\end{cases}$ $\Rightarrow$ $\begin{cases}\text{beginpunt }(-3,-7)\\D_g=[-3,\rightarrow\rangle\\B_g=[-7,\rightarrow\rangle\end{cases}$

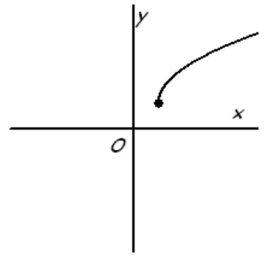
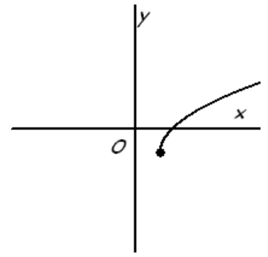
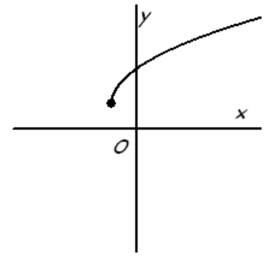
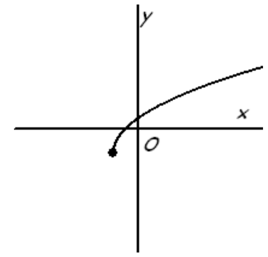
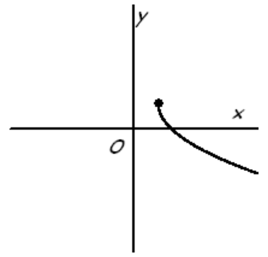
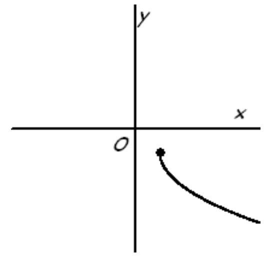
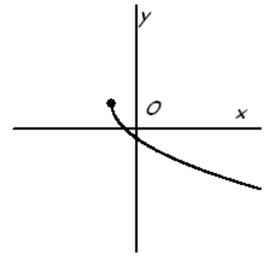
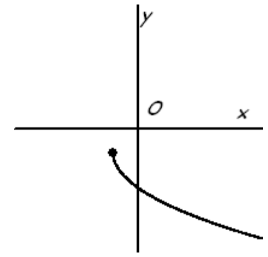
59c $y=\sqrt{x}$ $\xrightarrow{\text{verm. }x\text{-as, }-2}$ $y=-2\sqrt{x}$ $\xrightarrow{\text{tr. }(-1,0)}$ $h(x)=-2\sqrt{x+1}$.
$\begin{cases}\text{beginpunt }(0,0)\\D=[0,\rightarrow\rangle\\B=[0,\rightarrow\rangle\end{cases}$ $\Rightarrow$ $\begin{cases}\text{beginpunt }(0,0)\\D=[0,\rightarrow\rangle\\B=\langle\leftarrow,0]\end{cases}$ $\Rightarrow$ $\begin{cases}\text{beginpunt }(-1,0)\\D_h=[-1,\rightarrow\rangle\\B_h=\langle\leftarrow,0]\end{cases}$

59d $y=\sqrt{x}$ $\xrightarrow{\text{verm. }x\text{-as, }3}$ $y=3\sqrt{x}$ $\xrightarrow{\text{tr. }(0,1)}$ $k(x)=3\sqrt{x}+1$.
$\begin{cases}\text{beginpunt }(0,0)\\D=[0,\rightarrow\rangle\\B=[0,\rightarrow\rangle\end{cases}$ $\Rightarrow$ $\begin{cases}\text{beginpunt }(0,0)\\D=[0,\rightarrow\rangle\\B=[0,\rightarrow\rangle\end{cases}$ $\Rightarrow$ $\begin{cases}\text{beginpunt }(0,1)\\D_k=[0,\rightarrow\rangle\\B_k=[1,\rightarrow\rangle\end{cases}$

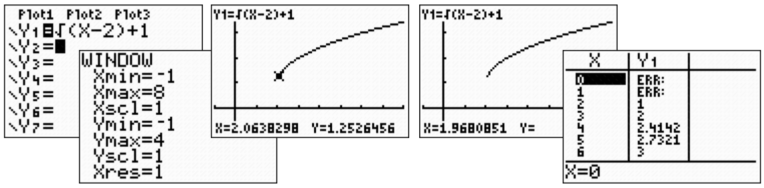
59e $y=\sqrt{x}$ $\xrightarrow{\text{verm. }x\text{-as, }-1}$ $y=-\sqrt{x}$ $\xrightarrow{\text{tr. }(1,-1)}$ $l(x)=-\sqrt{x-1}+1$.
$\begin{cases}\text{beginpunt }(0,0)\\D=[0,\rightarrow\rangle\\B=[0,\rightarrow\rangle\end{cases}$ $\Rightarrow$ $\begin{cases}\text{beginpunt }(0,0)\\D=[0,\rightarrow\rangle\\B=\langle\leftarrow,0]\end{cases}$ $\Rightarrow$ $\begin{cases}\text{beginpunt }(1,-1)\\D_l=[1,\rightarrow\rangle\\B_l=\langle\leftarrow,-1]\end{cases}$

59f $y=\sqrt{x}$ $\xrightarrow{\text{tr. }(0,-3)}$ $m(x)=\sqrt{x}-3$.
$\begin{cases}\text{beginpunt }(0,0)\\D=[0,\rightarrow\rangle\\B=[0,\rightarrow\rangle\end{cases}$ $\Rightarrow$ $\begin{cases}\text{beginpunt }(0,-3)\\D_m=[0,\rightarrow\rangle\\B_m=[-3,\rightarrow\rangle\end{cases}$

60

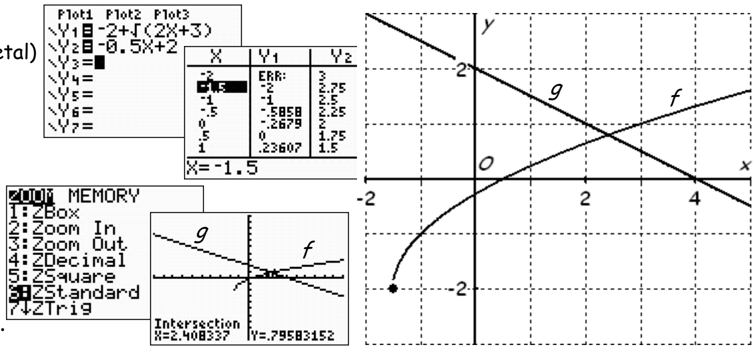
	$f(x)=a\sqrt{x-p}+q$			
	$p>0$		$p<0$	
	$q>0$	$q<0$	$q>0$	$q<0$
$a>0$	 $D=[p,\rightarrow\rangle$ en $B=[q,\rightarrow\rangle$	 $D=[p,\rightarrow\rangle$ en $B=[q,\rightarrow\rangle$	 $D=[p,\rightarrow\rangle$ en $B=[q,\rightarrow\rangle$	 $D=[p,\rightarrow\rangle$ en $B=[q,\rightarrow\rangle$
$a<0$	 $D=[p,\rightarrow\rangle$ en $B=\langle\leftarrow,q]$	 $D=[p,\rightarrow\rangle$ en $B=\langle\leftarrow,q]$	 $D=[p,\rightarrow\rangle$ en $B=\langle\leftarrow,q]$	 $D=[p,\rightarrow\rangle$ en $B=\langle\leftarrow,q]$

61a $y=\sqrt{x}$ $\xrightarrow{\text{tr. }(2,1)}$ $f(x)=\sqrt{x-2}+1$.
beginpunt $(0,0)$ $\Rightarrow$ beginpunt $(2,1)$



61b De GR geeft (2,06; 1,25) als beginpunt.
Dat komt omdat de GR bij TRACE
met een zekere stapgrootte werkt.

62a $2x+3\geq 0$ (onder het $\sqrt{\ }$-teken nooit een negatieve getal)
$2x\geq -3$
$x\geq -1{,}5\Rightarrow D_f=[-1{,}5;\rightarrow\rangle$.
$f(-1{,}5)=-2+\sqrt{0}=-2\Rightarrow$
beginpunt $(-1{,}5;\ -2)$.
(zie de grafiek hiernaast maak een tabel op de GR)



62b $f(x)=-2+\sqrt{...}\geq -2\Rightarrow B_f=[-2,\rightarrow\rangle$.

62c $f(x)=g(x)$ (intersect) $\Rightarrow x\approx 2{,}41$.
$f(x)<g(x)$ (zie plot en domein) $\Rightarrow -1{,}5\leq x<2{,}41$.

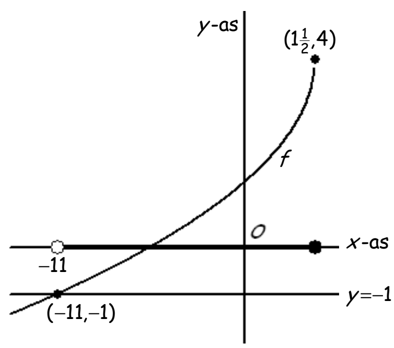
63a $8-4x\geq 0\Rightarrow -4x\geq -8\Rightarrow x\leq 2\Rightarrow D_f=\langle\leftarrow,2]$.
$f(x)=3+\sqrt{...}\geq 3\Rightarrow B_f=[3,\rightarrow\rangle$.
Het beginpunt is (2, 3).

63b $4x-8\geq 0\Rightarrow 4x\geq 8\Rightarrow x\geq 2\Rightarrow D_g=[2,\rightarrow\rangle$.
$g(x)=3+\sqrt{...}\geq 3\Rightarrow B_g=[3,\rightarrow\rangle$.
Het beginpunt is (2, 3).

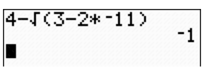
63c $2x+6\geq 0\Rightarrow 2x\geq -6\Rightarrow x\geq -3\Rightarrow D_h=[-3,\rightarrow\rangle$.
$h(x)=5-\sqrt{...}\leq 5\Rightarrow B_h=\langle\leftarrow,5]$.
Het beginpunt is (−3, 5).

63d $x\geq 0\Rightarrow D_k=[0,\rightarrow\rangle$.
$k(x)=3-2\sqrt{...}\leq 3\Rightarrow B_k=\langle\leftarrow,3]$.
Het beginpunt is (0, 3).

64ab $3-2x\geq 0\Rightarrow -2x\geq -3\Rightarrow x\leq \frac{3}{2}\Rightarrow D_f=\langle\leftarrow,1\frac{1}{2}]$.
$f(x)=4-\sqrt{...}\leq 4\Rightarrow B_f=\langle\leftarrow,4]$.
Het beginpunt is $(1\frac{1}{2},4)$. (ik beperk me hiernaast tot een schets van f)

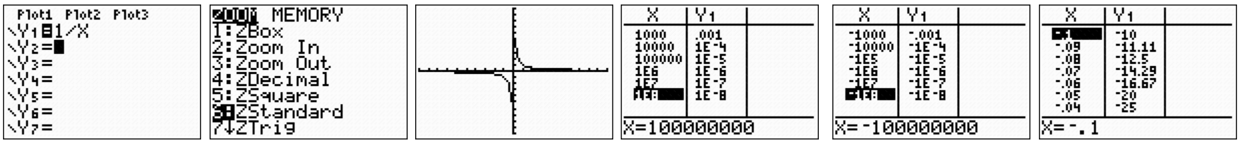


64c $4-\sqrt{3-2x}=-1$
$5=\sqrt{3-2x}$ (kwadrateren)
$25=3-2x$
$22=-2x$
$-11=x$ (voldoet).



$f(x)>-1\Rightarrow -1<f(x)\leq 4$. (zie bereik)
$-11<x\leq 1\frac{1}{2}$. (zie berekening en domein)

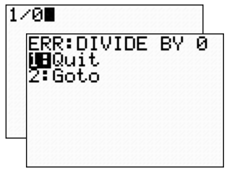
65a Zie de plot hieronder.



65b Neem je x achtereenvolgens 1000, 10 000, 100 000, ... dan komt $f(x)$ steeds dichter bij 0.
Neem je x achtereenvolgens -1000, $-10\,000$, $-100\,000$, ... dan komt $f(x)$ steeds dichter bij 0.

65c Als je x (negatief of positief) dicht bij 0 kiest, wordt $f(x)$ oneindig groot (negatief of positief).

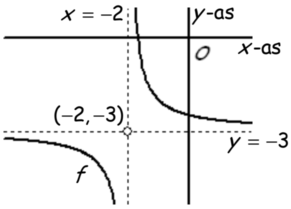
65d $\frac{1}{0} = ...,$ want $... \times 0 = 1$ (kan niet, want steeds is $... \times 0 = 0$).



66a $y = \frac{1}{x} \xrightarrow{\text{tr. }(-2,-3)} f(x) = \frac{1}{x+2} - 3.$

66b $\begin{cases}\text{V.A.: } x = 0 \\ \text{H.A.: } y = 0\end{cases} \Rightarrow \begin{cases}\text{V.A.: } x = -2 \\ \text{H.A.: } y = -3\end{cases}$

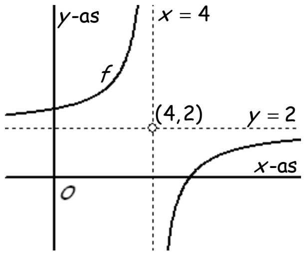
Zie de schets hiernaast. (vermeld de asymptoten!!!)



67a $y = \frac{1}{x} \xrightarrow{\text{verm. } x\text{-as, } -3} y = \frac{-3}{x} \xrightarrow{\text{tr. }(4,2)} f(x) = \frac{-3}{x-4} + 2.$

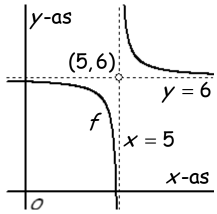
67b $\begin{cases}\text{V.A.: } x = 0 \\ \text{H.A.: } y = 0\end{cases} \Rightarrow \begin{cases}\text{V.A.: } x = 0 \\ \text{H.A.: } y = 0\end{cases} \Rightarrow \begin{cases}\text{V.A.: } x = 4 \\ \text{H.A.: } y = 2\end{cases}$

(vergeet in de schets niet de asymptoten te stippelen!!!)



68a $y = \frac{1}{x} \xrightarrow{\text{tr. }(5,6)} f(x) = \frac{1}{x-5} + 6.$

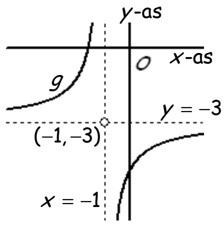
$\begin{cases}\text{V.A.: } x = 0 \\ \text{H.A.: } y = 0\end{cases} \Rightarrow \begin{cases}\text{V.A.: } x = 5 \\ \text{H.A.: } y = 6\end{cases}$



68b $y = \frac{1}{x} \xrightarrow{\text{verm. } x\text{-as, } -2} y = \frac{-2}{x} \xrightarrow{\text{tr. }(-1,-3)} g(x) = \frac{-2}{x+1} - 3.$

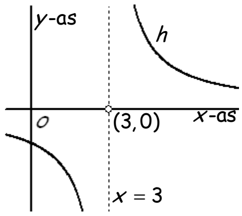
$\begin{cases}\text{V.A.: } x = 0 \\ \text{H.A.: } y = 0\end{cases} \Rightarrow \begin{cases}\text{V.A.: } x = 0 \\ \text{H.A.: } y = 0\end{cases} \Rightarrow \begin{cases}\text{V.A.: } x = -1 \\ \text{H.A.: } y = -3\end{cases}$

(in je schets de asymptoten gestippeld?)



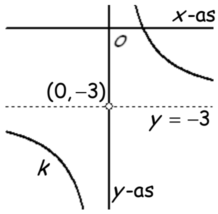
68c $y = \frac{1}{x} \xrightarrow{\text{verm. } x\text{-as, } 4} y = \frac{4}{x} \xrightarrow{\text{tr. }(3,0)} h(x) = \frac{4}{x-3}.$

$\begin{cases}\text{V.A.: } x = 0 \\ \text{H.A.: } y = 0\end{cases} \Rightarrow \begin{cases}\text{V.A.: } x = 0 \\ \text{H.A.: } y = 0\end{cases} \Rightarrow \begin{cases}\text{V.A.: } x = 3 \\ \text{H.A.: } y = 0\end{cases}$



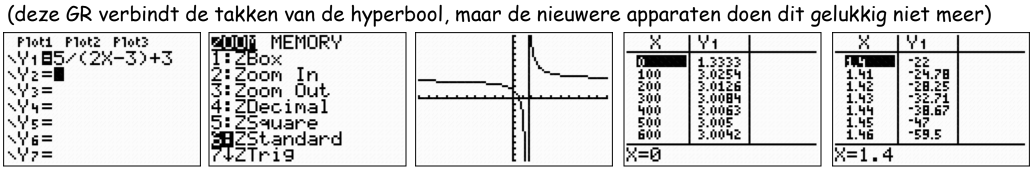
68d $y = \frac{1}{x} \xrightarrow{\text{verm. } x\text{-as, } 4} y = \frac{4}{x} \xrightarrow{\text{tr. }(0,-3)} k(x) = \frac{4}{x} - 3.$

$\begin{cases}\text{V.A.: } x = 0 \\ \text{H.A.: } y = 0\end{cases} \Rightarrow \begin{cases}\text{V.A.: } x = 0 \\ \text{H.A.: } y = 0\end{cases} \Rightarrow \begin{cases}\text{V.A.: } x = 0 \\ \text{H.A.: } y = -3\end{cases}$



69 Het schema is (wegens plaatsgebrek) afgedrukt na opgave 71.

70a Zie de plot hieronder.
(deze GR verbindt de takken van de hyperbool, maar de nieuwere apparaten doen dit gelukkig niet meer)



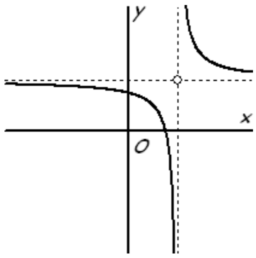
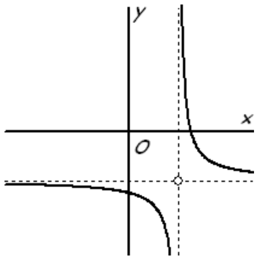
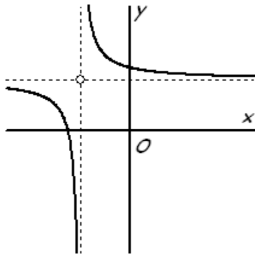
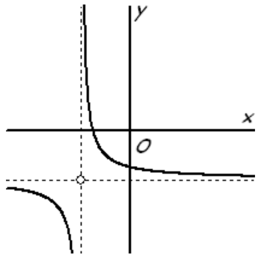
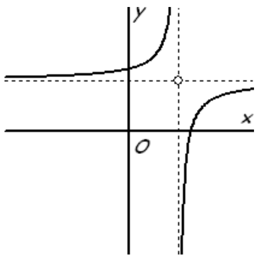
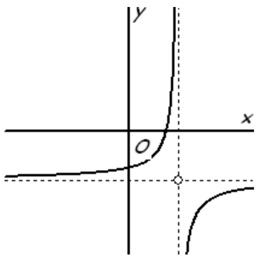
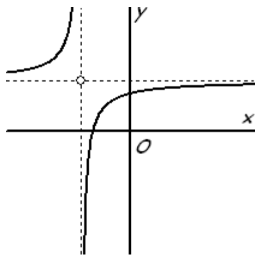
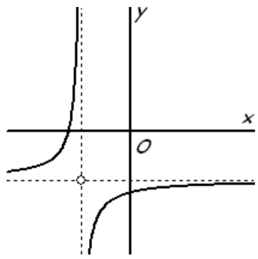
70b Neem je x achtereenvolgens 0, 100, 200, 300, ... dan komt $f(x)$ steeds dichter bij 3.
De horizontale asymptoot van de grafiek van f is de lijn $y = 3$.

70c Als je x dicht bij 1,5 kiest (eronder of erboven), wordt $f(x)$ oneindig groot (negatief of positief).

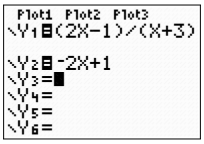
71a Noemer $= 0$ geeft $4 - x = 0 \Rightarrow -x = -4 \Rightarrow$ V.A.: $x = 4$.
Voor grote x is $f(x) = \frac{3x}{4-x} + 2 \approx \frac{3x}{-x} + 2 = -3 + 2 = -1 \Rightarrow$ H.A.: $y = -1$.

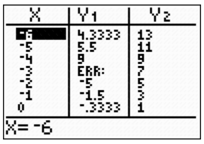
71b Noemer $= 0$ geeft $5 + 2x = 0 \Rightarrow 2x = -5 \Rightarrow x = -\frac{5}{2} \Rightarrow$ V.A.: $x = -2\frac{1}{2}$.
Voor grote x is $f(x) = \frac{2x-3}{5+2x} \approx \frac{2x}{2x} = 1 \Rightarrow$ H.A.: $y = 1$.

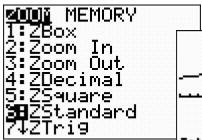
69

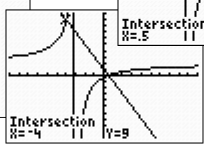
	$f(x)=\frac{a}{x-p}+q$			
	$p>0$		$p<0$	
	$q>0$	$q<0$	$q>0$	$q<0$
$a>0$	 V.A.: $x=p$ en H.A.: $y=q$.	 V.A.: $x=p$ en H.A.: $y=q$.	 V.A.: $x=p$ en H.A.: $y=q$.	 V.A.: $x=p$ en H.A.: $y=q$.
$a<0$	 V.A.: $x=p$ en H.A.: $y=q$.	 V.A.: $x=p$ en H.A.: $y=q$.	 V.A.: $x=p$ en H.A.: $y=q$.	 V.A.: $x=p$ en H.A.: $y=q$.

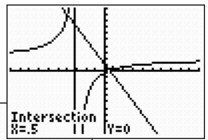
72a Noemer $=0$ geeft $x+3=0 \Rightarrow$ V.A.: $x=-3$.
Voor grote x is $f(x)=\frac{2x-1}{x+3}\approx\frac{2x}{x}=2 \Rightarrow$ H.A.: $y=2$.

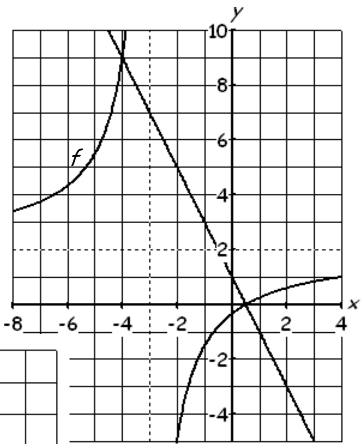






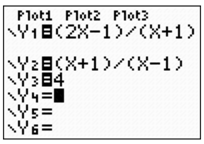


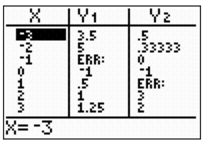


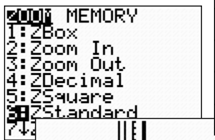


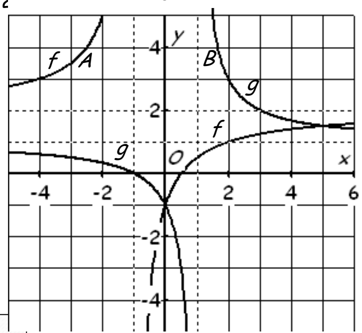
72b $\frac{2x-1}{x+3}=-2x+1$ (exact of intersect) $\Rightarrow x=-4 \ \vee \ x=\frac{1}{2}$.
$\frac{2x-1}{x+3}\le -2x+1$ (zie plot en domein) $\Rightarrow x\le -4 \ \vee \ -3<x\le\frac{1}{2}$.

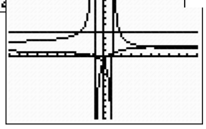
73a Noemer $=0$ geeft $x+1=0 \Rightarrow$ V.A.: $x=-1$.
Voor grote x is $f(x)=\frac{2x-1}{x+1}\approx\frac{2x}{x}=2 \Rightarrow$ H.A.: $y=2$.
Noemer $=0$ geeft $x-1=0 \Rightarrow$ V.A.: $x=1$.
Voor grote x is $g(x)=\frac{x+1}{x-1}\approx\frac{x}{x}=1 \Rightarrow$ H.A.: $y=1$.







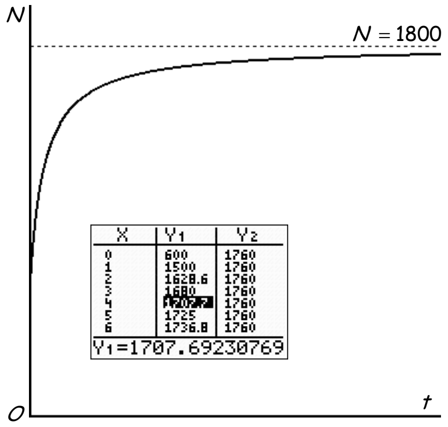




73b $\frac{2x-1}{x+1}=\frac{x+1}{x-1}$ (kruiselings vermenigvuldigen)
(exact, dus niet met intersect !!!)
$(2x-1)\cdot(x-1)=(x+1)\cdot(x+1)$
$2x^2-2x-x+1=x^2+x+x+1$
$x^2-5x=0$
$x\cdot(x-5)=0$
$x=0$ (voldoet) $\vee$ $x=5$ (voldoet).
$f(x)<g(x)$ (zie grafiek/plot en domein)
$-1<x<0 \ \vee \ 1<x<5$.

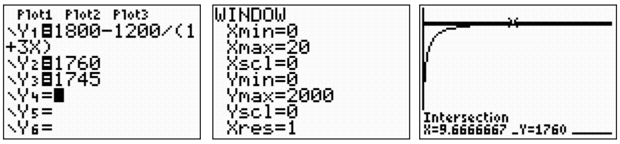
73c $\frac{2x-1}{x+1}=\frac{4}{1}$ (kruiselings vermenigvuldigen)
(exact, dus niet met intersect !!!)
$4\cdot(x+1)=1\cdot(2x-1)$
$4x+4=2x-1$
$2x=-5$
$x=-2{,}5$ (voldoet en $y=4$) $\Rightarrow A(-2{,}5;\ 4)$.
$\frac{x+1}{x-1}=\frac{4}{1}$
$4\cdot(x-1)=1\cdot(x+1)$
$4x-4=x+1$
$3x=5$
$x=\frac{5}{3}$ (voldoet en $y=4$) $\Rightarrow B(\frac{5}{3},4)$.
$AB=x_B-x_A=\frac{5}{3}--2\frac{1}{2}=4\frac{1}{6}$.

74a Voor grote x is $N(t)=1800-\frac{1200}{1+3t}\approx 1800 \Rightarrow$ H.A.: $N=1800$.
(dit betekent dat het aantal insecten niet boven de 1800 uitkomt)



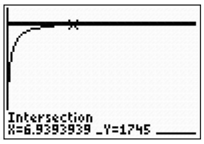
74b Zie de schets hiernaast. (stippel de horizontale symptoot)

74c $N(t)=1800-\frac{1200}{1+3t}=1760$ (intersect) $\Rightarrow t\approx 9{,}7$.
Dus op de tiende dag zijn er 1760 insecten.

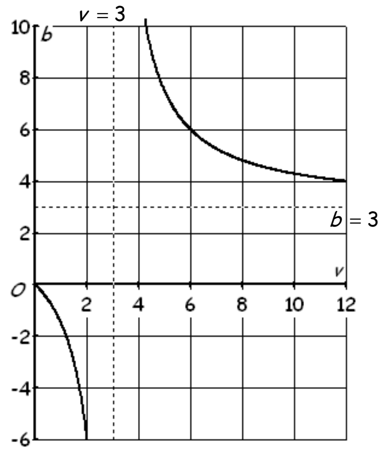


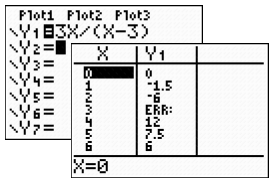
74d De vierde dag loopt van $t=3$ tot $t=4$.
$N(3)=1680$ en $N(4)\approx 1707$. (TABLE)
Er zijn de vierde dag $1707-1680=27$ insecten bijgekomen.

74e $N(t)=1800-\frac{1200}{1+3t}=1680 \Rightarrow t=3$. (zie 74d)
$N(t)=1800-\frac{1200}{1+3t}=1745$ (intersect) $\Rightarrow t\approx 6{,}9$.
Het duurt dus ongeveer $7-3=4$ dagen.



75a $f=3$ geeft: $\frac{1}{3}=\frac{1}{b}+\frac{1}{v}$
$\frac{1}{3}-\frac{1}{v}=\frac{1}{b}$
$\frac{1}{3}\cdot\frac{v}{v}-\frac{1}{v}\cdot\frac{3}{3}=\frac{1}{b}$
$\frac{v-3}{3v}=\frac{1}{b}$ (kruiselings vermenigvuldigen of breuken omkeren)
$b\cdot(v-3)=3v$ of $\frac{3v}{v-3}=\frac{b}{1}$
$b=\frac{3v}{v-3}$ $\frac{3v}{v-3}=b$.





75b $b=\frac{3v}{v-3}$. (de grafiek staat hiernaast)
Noemer $=0$ geeft $v-3=0 \Rightarrow$ V.A.: $v=3$.
Praktische betekenis: als de voorwerpsafstand $v=3$ dan is er geen beeld.
Voor (oneidig) grote v is $b=\frac{3v}{v-3}\approx\frac{3v}{v}=3 \Rightarrow$ H.A.: $b=3$.
Prakt. betekenis: er is geen voorwerpsafstand waarvoor de beeldpuntsafstand $b=3$. (voorwerp staat oneindig ver weg)

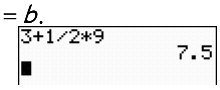
75c $b=v$ geeft
$v=\frac{v}{1}=\frac{3v}{v-3}$ (kruiselings vermenigvuldigen)
(exact, dus niet met intersect !!!)
$v\cdot(v-3)=1\cdot 3v$
$v^2-3v=3v$
$v^2-6v=0$
$v\cdot(v-6)=0$
$v=0 \vee v=6$.
($v=0$ voldoet niet omdat een noemer in lenzenformule nul wordt)
Dus bij $v=6$ is $b=v=6$.

75d $\left|\frac{b}{v}\right|=2$ geeft
$\left|\frac{\frac{3v}{v-3}}{v}\right|=2$ (exact, dus niet met intersect !!!)
$\left|\frac{3}{v-3}\right|=2$
$\frac{3}{v-3}=2 \vee \frac{3}{v-3}=-2$ (kruiselings vermenigvuldigen)
$2\cdot(v-3)=3 \vee -2\cdot(v-3)=3$
$2v-6=3 \vee -2v+6=3$
$2v=9 \vee -2v=-3$
$v=4\frac{1}{2}$ (voldoet) $\vee$ $v=1\frac{1}{2}$ (voldoet).

Diagnostische toets

D1a k: $y=ax+b$ met $a=\text{rc}_k=2$.
$\left.\begin{array}{l} k: y=2x+b \\ \text{door } A(-1,\ 6)\end{array}\right\} \Rightarrow 6=2\cdot -1+b$
$8=b$.
Dus k: $y=2x+8$.

D1b l: $y=ax+b$ met $a=\text{rc}_m=-\frac{1}{2}$.
$\left.\begin{array}{l} l: y=-\frac{1}{2}x+b \\ \text{door } B(9,\ 3)\end{array}\right\} \Rightarrow 3=-\frac{1}{2}\cdot 9+b$
$7\frac{1}{2}=b$.
Dus l: $y=-\frac{1}{2}x+7\frac{1}{2}$.



D1c k: $y=ax+5$ door $A(-10,\ 0)$
$0=a\cdot -10+5$
$10a=5$
$a=\frac{1}{2}$.

D2a $y=ax+b$ met $a=\frac{\Delta y}{\Delta x}=\frac{-2-2}{3--5}=\frac{-4}{8}=-0{,}5$.
$\left.\begin{array}{l} y=-0{,}5x+b \\ \text{door } A(-5,\ 2)\end{array}\right\} \Rightarrow 2=-0{,}5\cdot -5+b$
$2=2{,}5+b$
$-0{,}5=b$.
Dus $y=-0{,}5x-0{,}5$.

D2b $y=ax+b$ met $a=\frac{\Delta y}{\Delta x}=\frac{135-60}{65-40}=\frac{75}{25}=3$.
$\left.\begin{array}{l} y=3x+b \\ \text{door } P(40,\ 60)\end{array}\right\} \Rightarrow 60=3\cdot 40+b$
$60=120+b$
$-60=b$.
Dus $y=3x-60$.

D3a $W = at + b$ met $a = \frac{\Delta W}{\Delta t} = \frac{2900 - 500}{12 - 4} = \frac{2400}{8} = 300.$

$\left.\begin{array}{l} W = 300t + b \\ \text{door } (4,\ 500) \end{array}\right\} \Rightarrow 500 = 300 \cdot 4 + b$

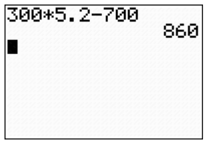
$500 = 1200 + b$

$-700 = b.$

Dus $W = 300t - 700.$

D3b $t = 5{,}2$ geeft

$W = 300 \cdot 5{,}2 - 700 = 860.$



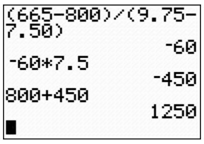
D4a $A = ap + b$ met $a = \frac{\Delta A}{\Delta p} = \frac{665 - 800}{9{,}75 - 7{,}50} = \frac{-135}{2{,}25} = -60.$

$\left.\begin{array}{l} A = -60p + b \\ \text{door } (7{,}5;\ 800) \end{array}\right\} \Rightarrow 800 = -60 \cdot 7{,}5 + b$

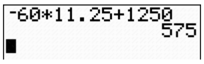
$800 = -450 + b$

$1250 = b.$

Dus $A = -60p + 1250.$



D4b $A = -60 \cdot 11{,}25 + 1250 = 575.$



D4c $A = 1000$ geeft

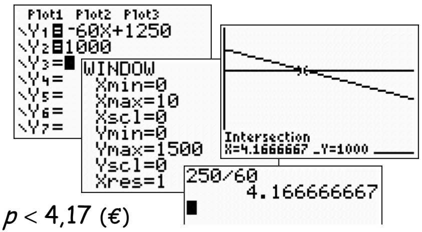
$1000 = -60p + 1250$

$-250 = -60p$

$\frac{-250}{-60} = p \approx 4{,}17$ (€).

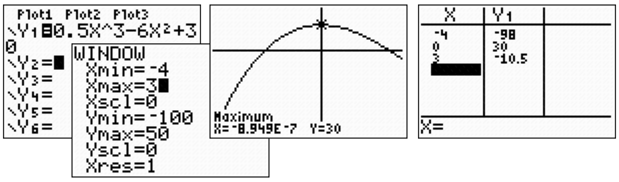
(mag ook met intersect)

$A > 1000$ (zie plot) $\Rightarrow p < 4{,}17$ (€)

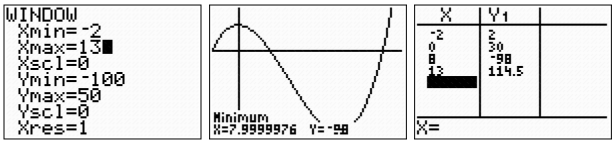


D5a $f(x) = 0{,}5x^3 - 6x^2 + 30$ heeft (opties in menu CALC) max. $f(0) = 30$ en min. $f(8) = -98.$ (zie D5b en D5c)

D5b $D_f = [-4,\ 3]$ (plot op dit domein) $\Rightarrow B_f = [-98,\ 30].$



D5c $D_f = [-2,\ 13]$ (plot op dit domein) $\Rightarrow B_f = [-98,\ 114\frac{1}{2}].$

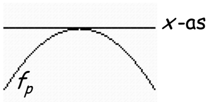


D6a $D = b^2 - 4ac = p^2 - 4 \cdot -1 \cdot -3 = 0$

$p^2 - 12 = 0$

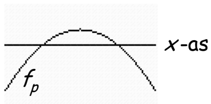
$p^2 = 12$

$p = -\sqrt{12} \ \vee \ p = \sqrt{12}.$



D6b $D = p^2 - 12$ ☺ > 0 (zie D6a)

$p < -\sqrt{12} \quad \vee \quad p > \sqrt{12}.$

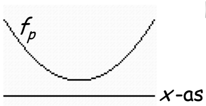


D7a $D = b^2 - 4ac = p^2 - 4 \cdot 1 \cdot 6p < 0$

$p^2 - 24p$ ☺ < 0

$p \cdot (p - 24) < 0$

$0 < p < 24.$



D7b $p^2 + p \cdot p + 6p = -4$

$2p^2 + 6p + 4 = 0$

$p^2 + 3p + 2 = 0$

$(p + 2) \cdot (p + 1) = 0$

$p = -2 \ \vee \ p = -1.$

D7c Voor de toppen geldt: $x = -\frac{b}{2a} = -\frac{p}{2 \cdot 1} = -\frac{p}{2}$ en

$y = f(x) = f(-\frac{p}{2}) = (-\frac{p}{2})^2 + p \cdot -\frac{p}{2} + 6p = \frac{1}{4}p^2 - \frac{1}{2}p^2 + 6p = -\frac{1}{4}p^2 + 6p = -13$ (keer -4).

$p^2 - 24p = 52$

$p^2 - 24p - 52 = 0$

$(p - 26) \cdot (p + 2) = 0$

$p = 26 \ \vee \ p = -2.$

D7d Voor de toppen geldt: $x = -\frac{p}{2}$ en $y = -\frac{1}{4}p^2 + 6p.$ (zie D7c)

De top $(-\frac{p}{2},\ -\frac{1}{4}p^2 + 6p)$ invullen in $y = 2x + 13$ geeft: $-\frac{1}{4}p^2 + 6p = -p + 13$ (keer -4)

$p^2 - 24p = 4p - 52$

$p^2 - 28p + 52 = 0$

$(p - 26) \cdot (p - 2) = 0$

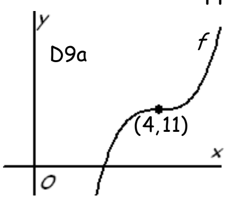
$p = 26 \ \vee \ p = 2.$

D8 Voor de x-coördinaat van de toppen geldt: $x = -\frac{b}{2a} = -\frac{-2p}{2 \cdot 1} = -p \Rightarrow p = -x.$

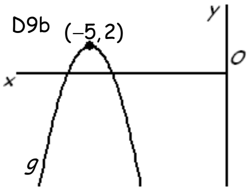
$y = f(x) \Rightarrow y = x^2 + 2 \cdot -x \cdot x - x = x^2 - 2x^2 - x = -x^2 - x.$ Dus de toppen liggen op $y = -x^2 - x.$

D9a $y = 2x^3$ ⇗ $\xrightarrow{\text{tr. } (4,11)}$ $f(x) = 2(x - 4)^3 + 11.$

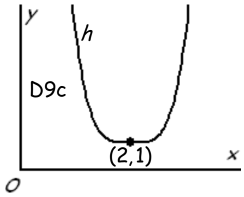
symm. in $(0,\ 0)$ $\Rightarrow$ symm. in $(4{,}11)$



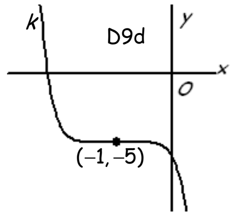
D9b $y=-3x^2$ ☹ $\xrightarrow{\text{tr. }(-5,2)}$ $g(x)=-3(x+5)^2+2.$
top (0, 0) $\Rightarrow$ top (−5, 2)



D9c $y=4x^4$ ☺ $\xrightarrow{\text{tr. }(2,1)}$ $h(x)=4(x-2)^4+1.$
top (0, 0) $\Rightarrow$ top (2,1)



D9d $y=-x^7$ ↘ $\xrightarrow{\text{tr. }(-1,-5)}$ $k(x)=-(x+1)^7-5.$
symm. in (0, 0) $\Rightarrow$ symm. in (−1, −5)



D10 $f(x)=2x^2$ ☺ $\xrightarrow{\text{tr. }(3,5)}$ $g(x)=2(x-3)^2+5.$

$\begin{cases}\text{top } (0,0)\\ \text{min. } f(0)=0\\ B=[0,\rightarrow\rangle\end{cases} \Rightarrow \begin{cases}\text{top } (3,5)\\ \text{min. } g(3)=5\\ B=[5,\rightarrow\rangle\end{cases}$

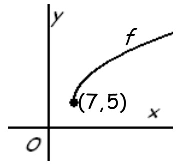
D11a $f(x)=\frac{1}{2}(x-2)^4+2$ ☺ $\xrightarrow{\text{verm. } x\text{-as, } 3}$ $y=1\frac{1}{2}(x-2)^4+6$ ☺ $\xrightarrow{\text{tr. }(3,-4)}$ $y=1\frac{1}{2}(x-5)^4+2$ ☺.
top (2, 2) $\Rightarrow$ top (2, 6) $\Rightarrow$ top (5, 2)

D11b $f(x)=\frac{1}{2}(x-2)^4+2$ ☺ $\xrightarrow{\text{tr. }(3,-4)}$ $y=\frac{1}{2}(x-5)^4-2$ ☺ $\xrightarrow{\text{verm. } x\text{-as, } 3}$ $y=1\frac{1}{2}(x-5)^4-6$ ☺.
top (2, 2) $\Rightarrow$ top (5, −2) $\Rightarrow$ top (5, −6)

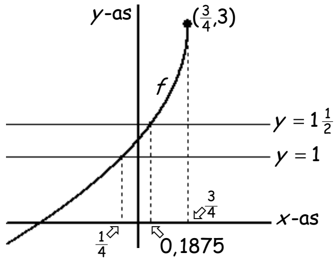
D12a $y=\sqrt{x}$ $\xrightarrow{\text{tr. }(7,5)}$ $f(x)=\sqrt{x-7}+5.$

D12c $\begin{cases}\text{beginpunt } (0,0)\\ D=[0,\rightarrow\rangle\\ B=[0,\rightarrow\rangle\end{cases} \Rightarrow \begin{cases}\text{beginpunt } (7,5)\\ D_f=[7,\rightarrow\rangle\\ B_f=[5,\rightarrow\rangle\end{cases}$

D12b Zie de schets hiernaast.

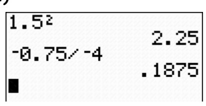


D13ab $3-4x\geq 0\Rightarrow -4x\geq -3\Rightarrow x\leq \frac{3}{4}\Rightarrow D_f=\langle\leftarrow,\ \frac{3}{4}].$
$f(x)=3-\sqrt{\ldots}\leq 3\Rightarrow B_f=\langle\leftarrow,3].$
Het beginpunt is $(\frac{3}{4},3)$. Gebruik een tabel voor het maken van de grafiek.
(ik beperk me hiernaast tot een schets van f)



D13c $3-\sqrt{3-4x}=1$
$2=\sqrt{3-4x}$ (kwadrateren)
$4=3-4x$
$1=-4x$
$-\frac{1}{4}=x$ (zie in schets).
$f(x)<1$ (gebruik de schets) $\Rightarrow x<-\frac{1}{4}$.

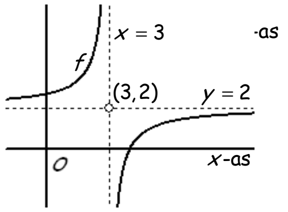
D13d $3-\sqrt{3-4x}=1{,}5$
$1{,}5=\sqrt{3-4x}$ (kwadrateren)
$2{,}25=3-4x$
$-0{,}75=-4x$
$0{,}1875=x$ (zie in schets)..



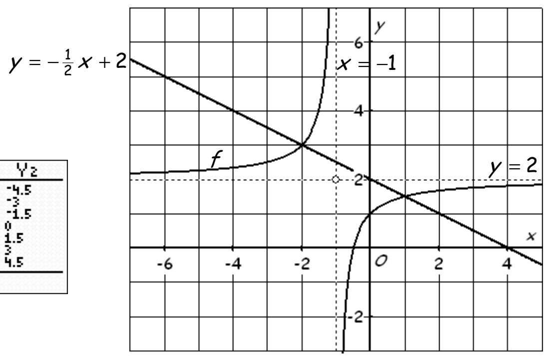
$f(x)>1{,}5$ (gebruik de schets) $\Rightarrow 0{,}1875<x\leq\frac{3}{4}$.

D14a $y=\frac{1}{x}$ $\xrightarrow{\text{verm. } x\text{-as, } -2}$ $y=\frac{-2}{x}$ $\xrightarrow{\text{tr. }(3,2)}$ $f(x)=\frac{-2}{x-3}+2.$

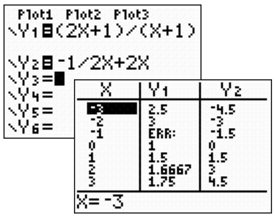
D14b $\begin{cases}\text{V.A.: } x=0\\ \text{H.A.: } y=0\end{cases} \Rightarrow \begin{cases}\text{V.A.: } x=0\\ \text{H.A.: } y=0\end{cases} \Rightarrow \begin{cases}\text{V.A.: } x=3\\ \text{H.A.: } y=2\end{cases}$
(vergeet niet de asymptoten te stippelen !!!)



D15a Noemer = 0 geeft $x+1=0\Rightarrow$ V.A.: $x=-1$.
Voor grote x is $f(x)=\frac{2x+1}{x+1}\approx\frac{2x}{x}=2\Rightarrow$ H.A.: $y=2$.



D15b $\frac{2x+1}{x+1}=-\frac{1}{2}x+2$ (kruiselings vermenigvuldigen)
(exact, dus niet met intersect !!!)



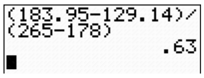
$1\cdot(2x+1)=(-\frac{1}{2}x+2)\cdot(x+1)$
$2x+1=-\frac{1}{2}x^2-\frac{1}{2}x+2x+2$
$\frac{1}{2}x^2+\frac{1}{2}x-1=0$
$x^2+1x-2=0$
$(x+2)\cdot(x-1)=0$
$x=-2\ \vee\ x=1.$ (voldoen)
$f(x)\geq-\frac{1}{2}x+2$ (zie grafiek/plot en domein) $\Rightarrow -2\leq x<-1\ \vee\ x\geq 1.$

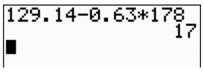
Gemengde opgaven 2. Functies en grafieken

G10a $l \,//\, m \Rightarrow rc_l = rc_m \Rightarrow a = -1\frac{1}{2}$; $A(0, 1)$ op $k \Rightarrow 1 = \frac{1}{2} \cdot 0 + 1$ (klopt); $A(0, 1)$ op $m \Rightarrow 1 = -1\frac{1}{2} \cdot 0 + b \Rightarrow 1 = b$.

G10b k snijden met de x-as ($y = 0$) $\Rightarrow 0 = \frac{1}{2}x + 1 \Rightarrow -1 = \frac{1}{2}x$ (keer 2) $\Rightarrow -2 = x$. Snijpunt $S(-2, 0)$.
$S(-2, 0)$ op $l \Rightarrow 0 = a \cdot -2 - 2 \Rightarrow 0 = -2a - 2 \Rightarrow 2a = -2 \Rightarrow a = -1$; S op $m \Rightarrow 0 = -1\frac{1}{2} \cdot -2 + b \Rightarrow 0 = 3 + b \Rightarrow -3 = b$.

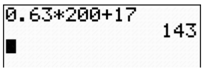
G10c $(4, 3)$ op $k \Rightarrow 3 = \frac{1}{2} \cdot 4 + 1$ (klopt).
$(4, 3)$ op $l \Rightarrow 3 = a \cdot 4 - 2 \Rightarrow 3 = 4a - 2 \Rightarrow 5 = 4a \Rightarrow \frac{5}{4} = a$; tevens $(4, 3)$ op $m \Rightarrow 3 = -1\frac{1}{2} \cdot 4 + b \Rightarrow 3 = -6 + b \Rightarrow 9 = b$.

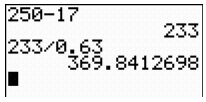
G11a $B = ax + b$ met $a = \frac{\Delta B}{\Delta x} = \frac{183{,}95 - 129{,}14}{265 - 178} = 0{,}63$.


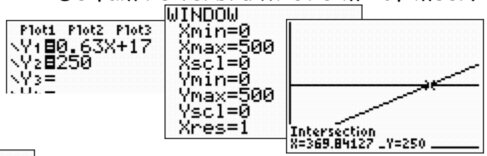
$B = 0{,}63x + b$
$x = 178 \ \wedge \ B = 129{,}14$ $\Rightarrow 129{,}14 = 0{,}63 \cdot 178 + b$
$17 = b$.


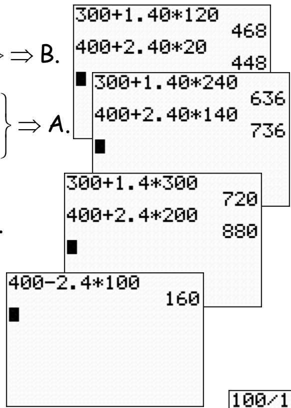
Dus $B = 0{,}63x + 17$.

G11b Het vastrecht is 17 (€).
De prijs per m^3 is 0,63 (€).

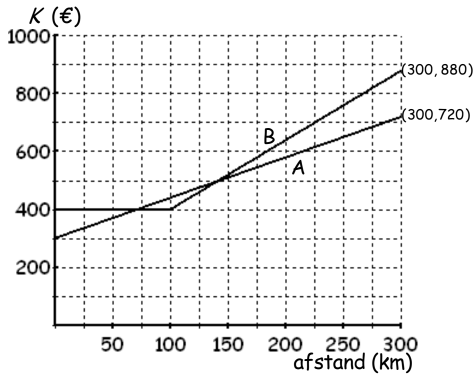
G11c $x = 200$ (m^3) geeft $B = 0{,}63 \cdot 200 + 17 = 143$ (€).


G11d $250 = 0{,}63x + 17$
$233 = 0{,}63x$
$x \approx 369{,}84$ (of intersect).


$0{,}63x + 17 > 250$ (zie plot) $\Rightarrow x > 369{,}8$.
De familie verbruikt 370 m^3 of meer.


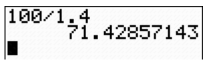
G12a Bij 120 km rekent A $300 + 1{,}40 \cdot 120 = 468$ (€)
en B $400 + 2{,}40 \cdot 20 = 448$ (€) $\Rightarrow$ B.
Bij 240 km rekent A $300 + 1{,}40 \cdot 240 = 636$ (€)
en B $400 + 2{,}40 \cdot 140 = 736$ (€) $\Rightarrow$ A.


G12b $K_A = 300 + 1{,}4x$.

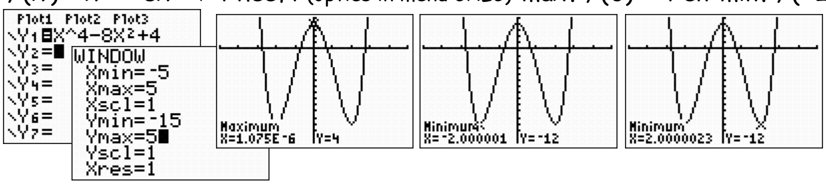
G12c Zie de grafieken in het assenstelsel hiernaast.


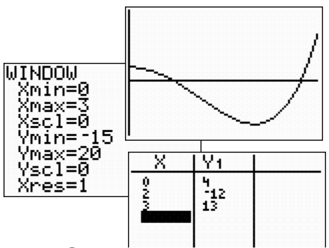
G12d $x > 100$: $K_B = 2{,}4x + b$
door $(100, 400)$ $\Rightarrow 400 = 2{,}4 \cdot 100 + b$
$160 = b$.

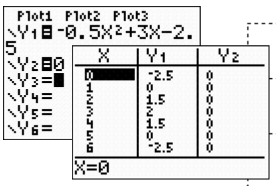
Dus $K_B = \begin{cases} 400 & \text{voor } x \le 100 \\ 2{,}4x + 160 & \text{voor } x > 100. \end{cases}$

G12e Voor $x \le 100$: $K_A = K_B \Rightarrow 300 + 1{,}4x = 400 \Rightarrow 1{,}4x = 100 \Rightarrow x \approx 71$.


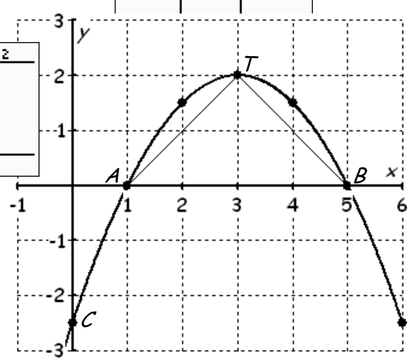
Voor $x > 100$: $K_A = K_B \Rightarrow 300 + 1{,}4x = 2{,}4x + 160 \Rightarrow 140 = x$. Dus bij (ongeveer) 71 km en 140 km.

G13a $f(x) = x^4 - 8x^2 + 4$ heeft (opties in menu CALC) max. $f(0) = 4$ en min. $f(-2) = -12$ en $f(2) = -12$.


G13b $D_f = [0, 3]$ (plot op dit domein) $\Rightarrow B_f = [-12, 13]$.


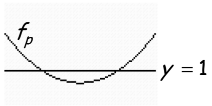
G14a Maak een mooie grafiek van deze functie. (zie hiernaast)


G14b $f(x) = -0{,}5x^2 + 3x - 2{,}5$ heeft (optie GR) max. $f(3) = 2$.
Dus top $T(3, 2)$ en bereik $B_f = \langle \leftarrow, 2]$ ($D_f = \mathbb{R}$).

G14c $f(0) = -2{,}5 \Rightarrow C(0;\ -2{,}5)$. (of met trace of met TABLE)
TC: $y = ax - 2{,}5$ door $T(3, 2) \Rightarrow 2 = 3a - 2{,}5 \Rightarrow 4{,}5 = 3a$.
TC: $y = 1{,}5x - 2{,}5$.


G14d $f(x) = -0{,}5x^2 + 3x - 2{,}5 = 0$ (keer -2)
$x^2 - 6x + 5 = 0$
$(x - 1) \cdot (x - 5) = 0$
$x = 1 \ \vee \ x = 5$.
$Opp_{\Delta ABT} = \frac{1}{2} AB \cdot h_T = \frac{1}{2} \cdot (5 - 1) \cdot 2 = 2 \cdot 2 = 4$.

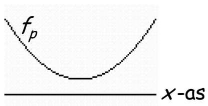
G15a $f_p(x) = x^2 + 4x + p = 1$ (moet 2 opl. hebben)
$x^2 + 4x + p - 1 = 0$ (moet 2 nulp. hebben)
$D = b^2 - 4ac = 4^2 - 4 \cdot 1 \cdot (p-1) > 0$
$16 - 4p + 4 > 0$
$-4p > -20$
$p < 5.$



of: voor de toppen geldt:
$x = -\frac{b}{2a} = -\frac{4}{2} = -2$ en
$y = f(-2) = (-2)^2 + 4 \cdot -2 + p = 4 - 8 + p = -4 + p < 1$
$-4 + p < 1$
$p < 5.$

G15b voor de toppen geldt: $x = -2$ en $y = -4 + p$ (zie de 2e uitwerking van G15a hierboven)
De top $(-2,\ -4+p)$ invullen in $y = 3x + 2 \Rightarrow -4 + p = -6 + 2 \Rightarrow p = 0.$

G16a $D = b^2 - 4ac = p^2 - 4 \cdot \frac{1}{2} \cdot 4 < 0$
$p^2 - 8$ ☺ < 0
$p^2 < 8$
$-\sqrt{8} < p < \sqrt{8}.$



G16b Voor de toppen geldt: $x = -\frac{b}{2a} = -\frac{p}{2 \cdot \frac{1}{2}} = -p$ en $y = f(x) = f(-p) = \frac{1}{2}(-p)^2 + p \cdot -p + 4 = \frac{1}{2}p^2 - p^2 + 4 = -\frac{1}{2}p^2 + 4.$

De top $(-p,\ -\frac{1}{2}p^2 + 4)$ invullen in $y = -5$ geeft:
$-\frac{1}{2}p^2 + 4 = -5$
$-\frac{1}{2}p^2 = -9$ (keer -2)
$p^2 = 18$
$p = -\sqrt{18} \ \vee \ p = \sqrt{18}.$

G16c De top $(-p,\ -\frac{1}{2}p^2 + 4)$ invullen in $y = -3x + 8$ geeft:
$-\frac{1}{2}p^2 + 4 = -3 \cdot -p + 8$
$-\frac{1}{2}p^2 - 3p - 4 = 0$ (keer -2)
$p^2 + 6p + 8 = 0$
$(p+4) \cdot (p+2) = 0$
$p = -4 \ \vee \ p = -2.$

G17 Voor de x-coördinaat van de toppen geldt: $x = -\frac{b}{2a} = -\frac{2p}{2 \cdot 1} = -p \Rightarrow p = -x.$
$y = f(x) \Rightarrow y = x^2 + 2 \cdot -x \cdot x + \frac{5}{-x} = x^2 - 2x^2 - \frac{5}{x} = -x^2 - \frac{5}{x}.$ Dus de toppen liggen op $y = -x^2 - \frac{5}{x}.$

G18a $f(x) = -\frac{1}{2}(x+2)^4 + 4$ ☹ $\xrightarrow{\text{verm. } x\text{-as, } 2}$ $y = -(x+2)^4 + 8$ ☹ $\xrightarrow{\text{tr. } (-2,-1)}$ $y = -(x+4)^4 + 7$ ☹.
top $(-2,\ 4)$ $\Rightarrow$ top $(-2,\ 8)$ $\Rightarrow$ top $(-4,\ 7)$

G18b $f(x) = -\frac{1}{2}(x+2)^4 + 4$ ☹ $\xrightarrow{\text{verm. } x\text{-as, } 4}$ $y = \dots$ ☹ $\xrightarrow{\text{tr. } (a,b)}$ $y = \dots$ ☹ $\begin{cases} -2 + a = 3 \\ 16 + b = 5 \end{cases} \Rightarrow \begin{cases} a = 5 \\ b = -11. \end{cases}$
top $(-2,\ 4)$ $\Rightarrow$ top $(-2,\ 16)$ $\Rightarrow$ top $(3,\ 5)$ $\Rightarrow$

G18c $f(x) = -\frac{1}{2}(x+2)^4 + 4$ ☹ $\xrightarrow{\text{tr. } (2,-6)}$ $y = \dots$ ☹ $\xrightarrow{\text{verm. } x\text{-as, } c}$ $y = \dots$
top $(-2,\ 4)$ $\Rightarrow$ top $(0,\ -2)$ $\Rightarrow$ top $(0,\ -2c)$.

G19a $f(x) = 0{,}25(x+2)^2 - 4$ ☺ $\xrightarrow{\text{tr. } (0,q)}$ $g(x) = 0{,}25(x+2)^2 - 4 + q$ ☺.
top $(-2,\ -4)$ $\Rightarrow$ top $(-2,\ -4+q)$
$g(x) = 0{,}25(x+2)^2 - 4 + q$ door $(0,\ 0) \Rightarrow 0{,}25 \cdot (0+2)^2 - 4 + q = 0 \Rightarrow 0{,}25 \cdot 4 - 4 + q = 0 \Rightarrow 1 - 4 = q \Rightarrow q = 3.$

G19b $f(x) = 0{,}25(x+2)^2 - 4$ ☺ $\xrightarrow{\text{tr. } (p,0)}$ $h(x) = 0{,}25(x - p + 2)^2 - 4$ ☺. Verder gaat de grafiek van h door $(0,\ 0)$ ⇓
top $(-2,\ -4)$ $\Rightarrow$ top $(-2+p,\ -4)$
$0{,}25 \cdot (-p+2)^2 - 4 = 0$
$0{,}25 \cdot (-p+2)^2 = 4$
$(-p+2)^2 = 16$
$-p + 2 = \pm 4.$
$-p = -2 \pm 4$
$p = 2 \mp 4$
$p = -2 \ \vee \ p = 6.$

G19c $f(x) = 0{,}25(x+2)^2 - 4$ ☺ $\xrightarrow{\text{verm. } x\text{-as, } a}$ $k(x) = 0{,}25a(x+2)^2 - 4a$ ☹.
$\begin{cases} \text{top } (-2,\ -4) \\ \text{min. } y(-2) = -4 \\ \text{B} = [-4,\ \rightarrow\rangle \end{cases} \Rightarrow \begin{cases} \text{top } (-2,\ -4a) \\ \text{max. } y(-2) = -4a \\ \text{B} = \langle \leftarrow,\ -4a] = \langle \leftarrow,\ 6] \end{cases}$
Dus $-4a = 6 \Rightarrow a = -1{,}5.$

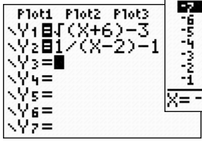
G19d Top $(-2, -4)$ op $m(x) = ax^4 + b \Rightarrow 16a + b = -4.$ ❶
$f(0) = 0{,}25 \cdot (0+2)^2 - 4 = 0{,}25 \cdot 4 - 4 = 1 - 4 = -3 \Rightarrow (0,\ -3)$ is snijpunt met y-as.
$(0,\ -3)$ op (de grafiek van) $m(x) = ax^4 + b \Rightarrow 0 + b = -3 \Rightarrow b = -3.$ ❷
❷ invullen in ❶ $\Rightarrow 16a - 3 = -4 \Rightarrow 16a = -1 \Rightarrow a = -\frac{1}{16}.$

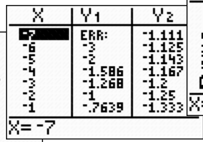
G20a $y=\sqrt{x} \xrightarrow{\text{tr. }(-6,-3)} f(x)=\sqrt{x+6}-3.$

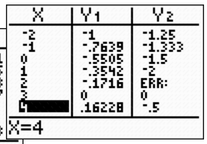
$\begin{cases}\text{beginpunt }(0,0)\\ D=[0,\rightarrow\rangle\\ B=[0,\rightarrow\rangle\end{cases} \Rightarrow \begin{cases}\text{beginpunt }(-6,-3)\\ D_f=[-6,\rightarrow\rangle\\ B_f=[-3,\rightarrow\rangle\end{cases}$

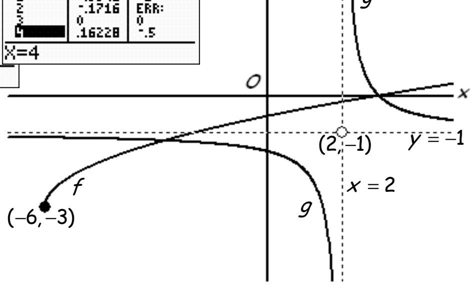
$y=\frac{1}{x} \xrightarrow{\text{tr. }(2,-1)} f(x)=\frac{1}{x-2}-1.$

$\begin{cases}\text{V.A.: } x=0\\ \text{H.A.: } y=0\end{cases} \Rightarrow \begin{cases}\text{V.A.: } x=2\\ \text{H.A.: } y=-1\end{cases}$



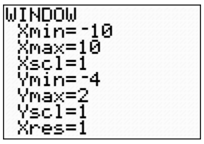


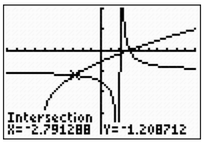


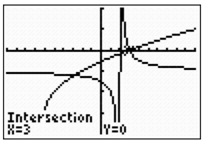


G20b $\sqrt{x+6}-3=\frac{1}{x-2}-1$ (intersect) $\Rightarrow x=-2{,}791 \vee x=3.$

$\sqrt{x+6}-3\le\frac{1}{x-2}-1$ (grafiek en domein) $\Rightarrow -6\le x\le -2{,}791 \vee -2<x\le 3.$





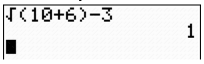


G20c $\sqrt{x+6}-3=1$ (exact, dus niet met intersect !!!)

$\sqrt{x+6}=4$ (kwadrateren)

$x+6=16$

$x=10$ (voldoet).



G20d $\frac{1}{x-2}-1=5$

$\frac{1}{x-2}=\frac{6}{1}$ (kruiselings vermenigvuldigen)

(exact, dus niet met intersect !!!)

$6\cdot(x-2)=1\cdot 1$

$6x-12=1$

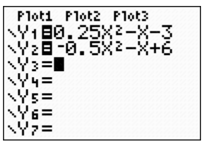
$6x=13$

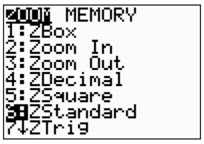
$x=\frac{13}{6}=2\frac{1}{6}$ (voldoet).

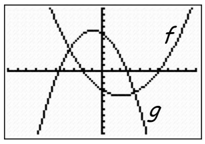
TI-84 3. Toppen

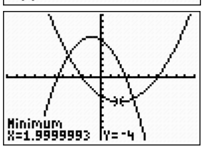
1a Zie de plot op $[-10,10]\times[-10,10]$ hiernaast.

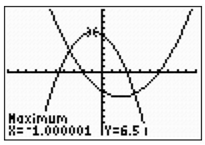
1b Optie minimum met [2nd] [TRACE] (=[CALC]) [3] $\Rightarrow$ top is $(2, -4)$.
(bij Left Bound? en Right Bound? met de cursor ([◄] of [►]), of door het intikken van een x-waarde, aan de juiste kant van de top gaan staan en [ENTER]; bij de vraag Guess? alleen nog maar [ENTER])
Optie maximum met [2nd] [TRACE] (=[CALC]) [4] $\Rightarrow$ top is $(-1, 6\frac{1}{2})$.
(kies de tweede formule na de optie maximum)











2a Zie de plot op $[-10,10]\times[-15,15]$ hiernaast.

2b De toppen zijn $(-2{,}79;\ 11{,}65)$ en $(4{,}79;\ -10{,}05)$.